Mathematics Mastery

At home materials

Guidance Pack

Year 4 Weeks 1-4

Pack 1: Numbers

Session A) Counting and grouping

Session B) Value of the place

Session C) Regrouping

Session D) Build and adjust

Pack 3: Multiplication facts

Session A) Multiplication facts

Session B) Doubling

Session C) Multiples of 10 and 5

Session D) Derived facts

Pack 4: Multiplication strategies

Session A) Adjusting a factor by 1

Session B) Monthly payments

Session C) Adjusting a factor by 10

Session D) Exploring calculation strategies

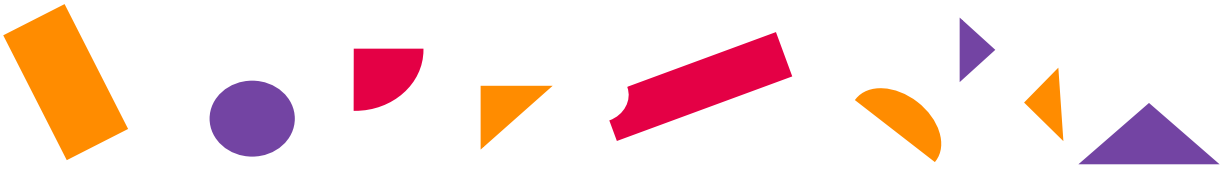
Pack 11: Division strategies

Session A) Division and multiplication

Session B) Halving strategies

Session C) Division structures

Session D) Models of division



Mathematics Mastery

Step-by-step

Timing

Each session is 30 minutes
20 minute Talk Task and 10 minute independent activity

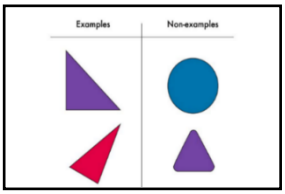
Session guidance

Get **them** talking and grow their language.

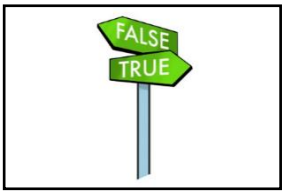
Get **them** to use equipment, manipulatives, models and images to show and explain.

Challenge **them** to think mathematically. Use the Prompts for Thinking listed below to help them to build up habits in the way they think about mathematical situations.

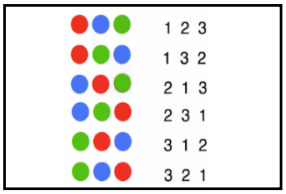
Reason it
Explain how you know. Focus on reasons rather than answers.
What could you say, do, draw or write to help someone else understand?



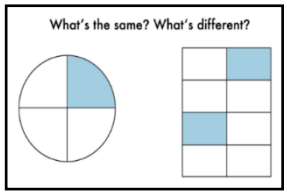
Generate examples and non-examples
What are the important features? What features are not important (e.g. colour)?



True or false?
If true, give examples to support your answer.
If false, give a counter example.



Find all possibilities
Have you found all the possible answers? How do you know? Did you work systematically?



What's the same? What's different?
Compare and contrast and look for connections.
How many different answers can you give?

Always, sometimes or never true?
Give examples to show if the statement is always, sometimes or never true. How do you know?

Mathematics Mastery

Step-by-step

Pack 1: Number

Session A: Counting and grouping

Resources needed: Dienes thousands, hundreds, tens and ones

The purpose of this session is to get pupils talking and thinking about numbers, what they can mean and how we write them. You want to explore what pupils understand about how our number system works.

Talk Task

Use the images to discuss numbers and think about where and why they are used. Is it to count, measure, label, order, …? Encourage pupils to include their own examples.

Ask pupils to think about how they would try to work out how many people there are in the school (adapt to a familiar place with lots of people if this isn't suitable for your setting). Going through the process of thinking about this will probably involve grouping the people in some way rather than thinking about each individual, e.g. there are __ teachers, there are __ children in each class,….

Connect this experience of grouping things in order to count them with the way we write numbers. Our number system uses grouping. Discuss what they know about our number system and how it works. We use 10 digits and with them we can write any number you can think of!

Ask pupils to count from zero and show with Dienes. When they reach ten what do they do? Continue to the number 13 and ask pupils to write the number down. How can we use the Dienes blocks to show what the digits mean? The blocks allow you to see the relationship between each place in the number system. That ten ones is equal to one ten. That ten tens is equal to one hundred.

Count and build from 88 to 111. Stop every now and again and think about how to record the number you are on in digits and written words. Connect the abstract digits to the Dienes and the spoken sounds.

Count in steps of 1, 10 or 100 from different starting points both forwards and backwards.

Activity

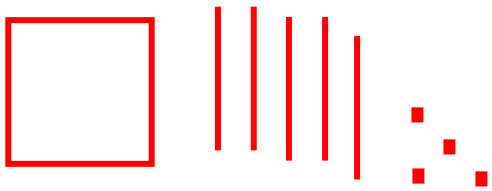
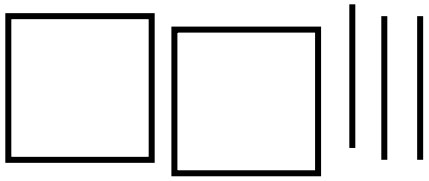
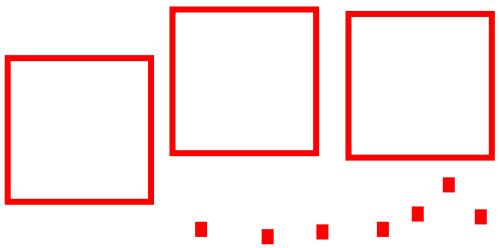
The activity prompts connection of different representations of number that focus on place value understanding. Pupils complete a table showing a number with digits, words and Dienes. They then consider counting in steps of ten starting from 56.

Video guidance

Activity: Counting and grouping

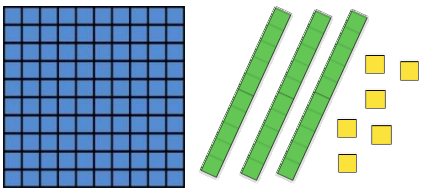
Answers

1) Complete the table to show each number with Dienes and in words.

number	Dienes	words
154		One hundred and fifty four
230		Two hundred and thirty
307		Three hundred and seven

2) If you count in steps of 10 starting at 56, will you say these numbers? Tick the ones you will say. What other numbers would you say?

Ninety six



106

Two hundred and twenty six

Any value greater than 56 with a 6 in the ones place

Mathematics Mastery

Step-by-step

Pack 1: Number

Session B: Value of the place

Resources needed: Dienes hundreds, tens and ones. Small pieces of paper.

The purpose of this task is to get pupils think about the fact that we use a place value number system: that the same digit can have a different value if it is in a different place.

Talk Task

Use three digits to explore how many different numbers can be made.

Examples are provided on the Talk Task sheet to prompt discussion. Build each number with Dienes, draw pictures of the Dienes and choose some numbers to write in words.

It would be useful to be able to write each number on small pieces of paper so they can be moved around, compared and ordered.

Explore 1-digit, 2-digit numbers and then 3-digit numbers. *How might we know if we have made all the possible numbers?*

Challenge pupils to think about how to explain that they have found them all. Encourage pupils to place them in order or group them by their starting (or first) digit to help convince themselves and you that there are no more options.

13, 14, 31, 34, 41, 43, 134, 143, 314, 341, 413, 431

Having made different numbers with the same digits, discuss and compare them focusing on how the value of the digit is different if it is in a different place.

In the number 134, the digit 4 has a value of 4 ones. In the number 143, the digit 4 has a value of 4 tens. In 413, the digit 4 has a value of 4 hundreds.

Sort the numbers in different ways. For example, odd and even, greater than 200 and less than 200, etc.

Extend the task by introducing a zero and exploring the options for the numbers that can be made.

Activity

The activity sheet guides students through similar experiences of using digits to write numbers and generating examples and non-examples of numbers with a given description.

Video guidance

Pack 1 Session B

Activity: The value of the place

Answers

1) Use these digits to create numbers for each of the properties

a) A number less than 100 — 54, 52, 45, 42, 24, 25

b) A number greater than 300 — 542, 524, 452, 425

c) An even number — 542, 524, 452, 254, 54, 52, 42, 24

d) A number that you can show with 7 Dienes blocks — 52, 25

e) An odd number — 425, 245, 45, 25

2) There are many ways to complete

	Examples	Non-examples
A number with 4 tens that is greater than 500	1420 3456	1320 425
An even number with 3 hundreds	346 346	325 458
A number with 6 ones that is greater than 100 but less than 200	106 196	195 206

Mathematics Mastery

Step-by-step

Pack 1: Number

Session C: Regrouping

Resources needed: Dienes (at least 2 hundreds, 13 tens, 13 ones)

The purpose of this session is to explore different ways the same number can be grouped. Being able to see numbers in lots of different ways supports being able to calculate flexibly.

Talk Task

There is a lot of information on the Talk Task sheet so fold it and look at each set of coins in turn. Discuss the relationships between the coins: that ten 1p coins is 10p, that £1 is the same as 100p or ten 10p coins.

For each set of coins, write down the value of each coin type in pence, discussing how you know and building models with Dienes to show and explain.

- 100p + 110p + 3p
- 200p + 13p
- 100p + 100p + 13p

Connect this experience to understanding our number system: that 10 ones is equal to 1 ten and that 10 tens is equal to 1 hundred.

Having looked at each set of coins, take time to look at them together and think about what is the same and what is different. Encourage pupils to think of as many different answers as they can.

The three sets of coins all have the same value of 213 pence or £2.13. None of these is the most efficient way of showing 213 pence. Ask pupils to show 213 pence with the fewest number of coins. Notice how it matches the written number with 2 hundreds, 1 ten and 3 ones

Extend the activity by thinking about other ways the number 213 can be grouped and calculations that can be written.

Activity
The activity sheet starts with the challenge of matching representations of three different numbers. Then pupils complete empty boxes in calculations. There are lots of patterns to find and extend within this task and you can encourage pupils to look for these. They should create more examples for each number. Extend the activity by selecting other numbers to explore.

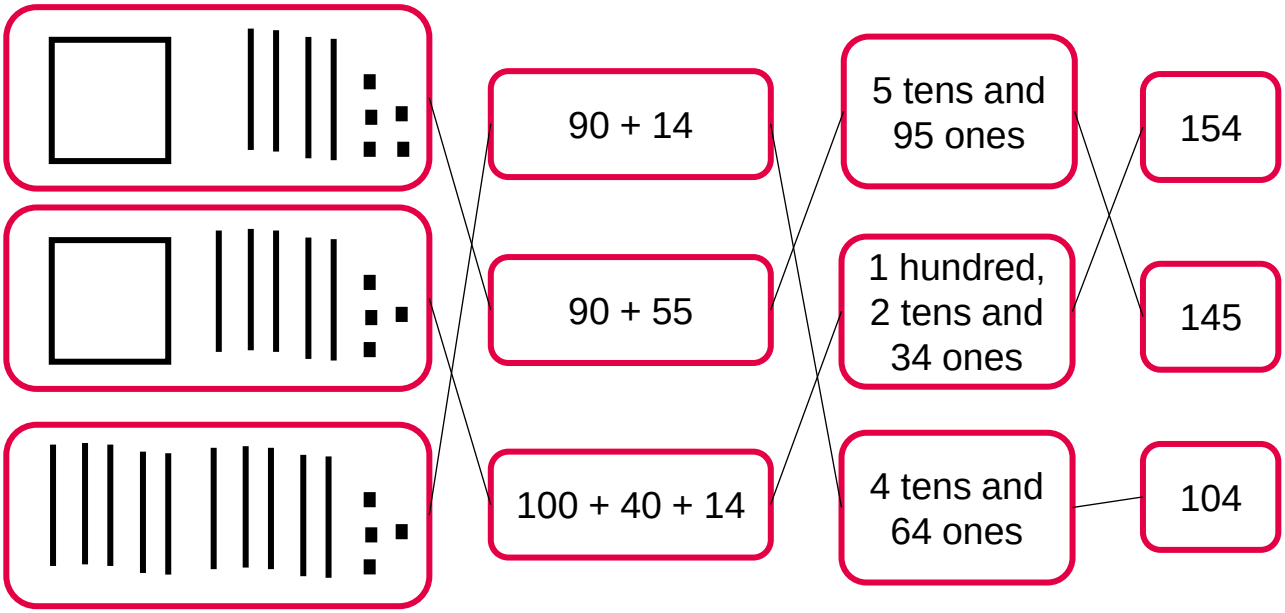
Video guidance

Pack 1 Session C

Activity: Regrouping

Answers

1) Match the representations



2) Fill in the blanks to show each number in different ways. How many more can you think of?

42	84	168
$40 + 2$	$80 + 4$	$100 + 60 + 8$
$30 + 12$	$60 + 24$	$100 + 50 + 18$
$20 + 22$	$50 + 34$	$100 + 40 + 28$
$21 + 21$	$51 + 33$	
		$90 + 70 + 8$
$10 + 32$	$30 + 54$	$90 + 60 + 18$
$33 + 9$	$20 + 64$	$90 + 50 + 28$
	$10 + 74$	
		$80 + 80 + 8$

Mathematics Mastery

Step-by-step

Pack 1: Number

Session D: Build and adjust

Resources needed: Dienes, 10 ones, 10 tens, 10 hundreds and 1 thousand

The purpose of this session is to play with numbers and think about what can and cannot happen when you restrict or adjust.

Talk Task

Use exactly ten Dienes blocks to build numbers and explore the different numbers that can be shown with 10 blocks. Choose from ones, tens or hundreds and extend to thousands if appropriate.

Record the numbers and images of the numbers. There are plenty of opportunities for finding and extending patterns when generating examples.

Remember to include non-examples by discussing the numbers that cannot be shown with exactly ten blocks.

Having generated lots of examples, choose a few examples and think about what could happen if you adjust the model. Add one more block and explore possible outcomes. Take away a block and explore possible outcomes.

Draw attention to which digits change and how they change to connect to the next section.

Discuss and explore how the digits change when 10 is added to a number. Which digit will always change, which will never change and which will sometimes change? Create examples to support conclusions.

The digit in the ones place will never change.

The digit in the tens place will always change.

The digit in the hundreds place will sometimes change.

Activity

The activity sheet guides students through similar tasks of creating numbers with five Dienes blocks and thinking about how the digits change when one is added.

The digit in the ones place always changes.

The digit in the tens place sometimes changes.

The digit in the hundreds place sometimes changes.

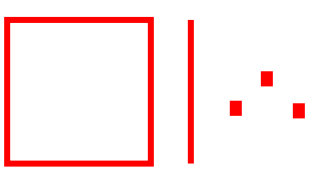
Video guidance

Pack 1 Session D

Activity: Build and adjust

Answers

1) Draw and write numbers with **exactly five Dienes blocks**



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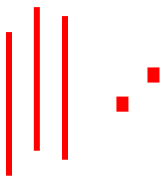


32

5, 14, 41, 23, 50
104, 122, 131, 140, 203, 212, 221, 230,
302, 320, 401, 410

2) Circle always, sometimes or never and give examples to support your answer.

always
sometimes
never

If you add 1 to a number, the digit in the ones place changes.
10+1=11, 19+1=20

always
sometimes
never

If you add 1 to a number, the digit in the tens place changes.
Changes: 39 + 1 = 40
Doesn't change: 38 + 1 = 39

always
sometimes
never

If you add 1 to a number, the digit in the hundreds place changes.
Changes: 199 + 1 = 200
Doesn't change: 234 + 1 = 235

Mathematics Mastery

Step-by-step

Pack 3: Multiplication facts

Session A: Multiplication facts

Resources needed: Multiplication fact sheet. Times tables poster (if available)

The purpose of this session is to explore and establish the current level of confidence with the multiplication facts. The following sessions focus on using what you know to work out related facts.

Activity

Complete the activity sheet first and refer to it during the Talk Task. In primary school, pupils are expected to know the multiplication facts up to 12 × 12. Ask pupils to think about the multiplication facts (or times tables) they know. Group the facts into those they have memorised, those they can work out quickly and those they find tricky.

Discuss and reflect on experiences with using and learning times tables facts. Keep this grid for reference as it would be useful to return to and track facts that change position.

Video guidance

Talk Task

Follow on from this activity using the talk mat which shows a multiplication grid containing all of the facts expected to be known by the end of primary school. Ask students to explain how to read the grid for both multiplication and division. Use the gird to show 6 × 8. Use the grid to show 35 ÷ 7.

Ask students to convince you that there are 169 facts on the grid.

Discuss that any number multiplied by zero is equal to zero. Discuss that a number multiplied by one is equal to the number and think of situation to describe this. For example, 1 bag of 7 apples is 7 apples, 7 × 1 = 7

Use the grid to continue discussion about which facts pupils know and which they find tricky. Encourage them to shade in known facts and think about if they know any relationships between facts.

Possible discussion points that will be built upon in later session in this and other packs:

Commutativity: If I know 3 groups of 5 then I know 5 groups of 3

Doubling: I can double my 3 times tables to get my 6 times tables

Combining facts: To work out 7 × 6, add 5 × 6 and 2 × 6

Adjusting facts: I can adjust the tens to get the nines. 7 × 9 is 7 less than 70

Mathematics Mastery

Step-by-step

Pack 3: Multiplication facts

Session B: Doubling

Resources needed: Cubes or other countable materials

The purpose of this session is to help pupils to realise that they can use facts they know and doubling to work out related facts.

Talk Task

Doubling is used throughout so start the session by asking pupils to use cubes to show you double and half of given amounts. *Use cubes to show double 8.*

The talk mat is designed to prompt discussion about the connection between the multiplication facts for 2 and 4. Prompt pupils to use cubes to explore the instructions and decide if it is always, sometimes or never true. Encourage them to try to explain the reasons for their answer using the cubes to show.

The images of cubes with the calculation give evidence to support understanding why doubling a number and doubling again is a multiple of 4. It is the same as multiplying the number by 4.

Conclude this by writing some facts from the 2 times tables and doubling them to work out facts in the 4 times tables so that pupils see how they can use this understanding.

The true or false task will need a lot of cubes to represent so instead use an area model. Encourage pupils to draw and label rectangles to help them show which calculations are the same as double 4 × 6.

Having discussed and concluded that 8 × 6 and 4 × 12 are equal to double 4 × 6, look at the numbers in calculations and describe relationships that can be seen. E.g. one factors is double, that other factor is the same, the product is double. This will be built upon in the activity sheet.

Extend the activity by asking pupils to think of other calculations that they can use doubling to work out.

Activity

The activity sheet consolidates the ideas explored in the Talk Task. Pupils write calculations to describe array of counters that are doubled.

Then they give evidence to support calculation strategies for dividing by 4 and multiplying by 8. For the dividing by 4 strategy you may want to give pupils the starting numbers 40, 24 or 88 to explore. Finally pupils match calculations to a doubling strategies and the answer.

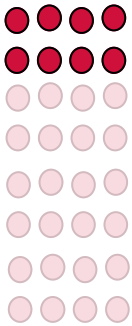
Video guidance

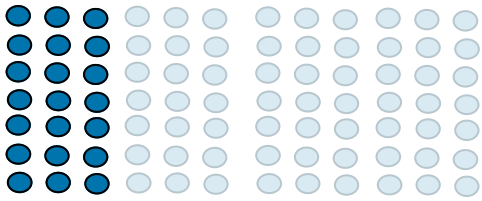
Pack 3 Session B

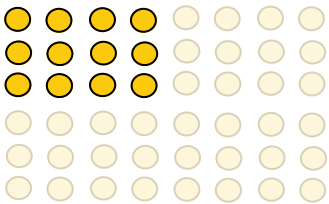
Answers

Activity: Equal groups

1) Use these arrays and doubling to complete the calculations







$4 \times 2 = 8$

$4 \times 4 = 16$

$4 \times 8 = 32$

$3 \times 7 = 21$

$6 \times 7 = 42$

$12 \times 7 = 84$

$3 \times 4 = 12$

$3 \times 8 = 24$

$6 \times 8 = 48$

2) Give examples to show that each of these strategies works.

$4 \times 2 = 8$

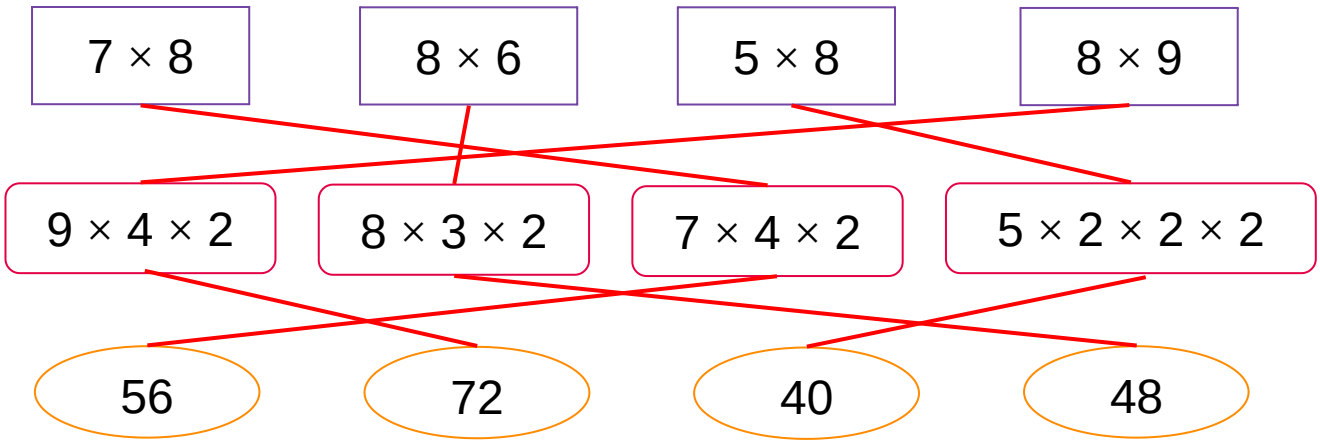
To divide a number by 4, I can halve twice

Example:
12 ÷ 4 = 3
Half of 12 is 3 and half of 6 is 3

To multiply a number by 8, I can double three times

Example:
5 x 8 = 40
Double 5 is 10, double 10 is 20 and double 20 is 40.

3) Match each calculation to a valid strategy and then to the answer.



Mathematics Mastery

Step-by-step

Pack 3: Multiplication facts

Session C: Multiples of 10 and 5

Resources needed: Dienes, a place value chart drawn on paper, bead string

The purpose of this session is to think in detail about how to explain multiplying by ten and then to explore the relationship between multiples of ten and multiples of five.

Talk Task

The first part of the Talk Task challenges students to think about how to explain multiplying by 10. Saying “just add 0” is not a good enough explanation. The language is not precise enough because 12+0 is not 120. The idea of adding a zero comes from noticing that all multiples of 10 have a zero in the ones place and this is a place to start. Why does this happen?

Ask pupils to use Dienes on a place value chart to show a number and then show that number multiplied by 10. Discuss our number system, using the blocks as tools to work with as they try to clearly explain how to multiply by 10.

Our number system is based on ten. Each place to the left is ten times greater. 10 ones is 1 ten, 10 tens is 1 hundred, … If all the digits are one place to the left then the number is ten times greater. The zero can be described as a place holder because it holds the place of the ones so the other digits have a value that is ten times greater.

Focusing on the place of each digit rather than ‘adding zero’ is more useful when working with decimal numbers because you cannot just put a zero on the end. For example, 1.4 × 10 is not 1.40

The second part of the Talk Task prompts pupils to use a bead string to explore a calculation strategy: ‘to multiply a number by 5 you can multiply by 10 and halve the result’.

Explore different starting numbers, noticing the doubling and halving relationship. Prompt pupils to give other calculations involving larger numbers to demonstrate their understanding of this relationship.

Activity

The activity sheet provides similar experiences, with models provided to prompt further thought about the connection between multiplying by 5 and multiplying by 10. Pupils go on to work with calculations outside of the times tables to calculate using this strategy.

For example, 26 × 5 = half of 260 = 130

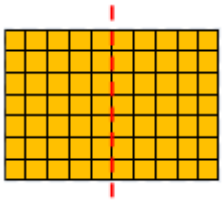
Video guidance

Pack 3 Session C
Activity: Regrouping

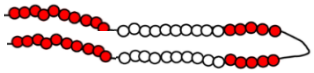
Answers

1) Write calculations to describe each model.

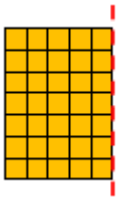
5 × 10 = 50



7 × 10 = 70

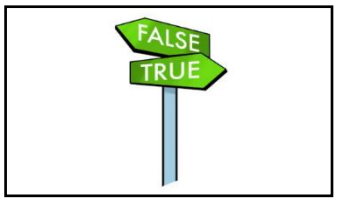


5 × 5 = 25



7 × 5 = 35

2) Decide if the following are true or false. If they are true, then calculate the answer. If they are false, give a correct statement and calculate the answer.



Half of 80 = 5 × 8	True
7 × 5 = half of 30	False, 7 × 5 = half of 70
12 × 5 = 6 × 10	True

3) Use the relationships between multiples of 10 and 5 to complete the calculations

$12 \times 10 = 120$ → find half → $60 = 12 \times 5$

$16 \times 10 = 160$ → find half → $80 = 16 \times 5$

$26 \times 10 = 260$ → find half → $130 = 26 \times 5$

Mathematics Mastery

Step-by-step

Pack 3: Multiplication facts

Session D: Derived facts

Resources needed: Counters and a whiteboard pen to write values onto them

The purpose of this task is to realise how much can be worked out from a known multiplication fact by using multiplication by 10, 100, …

Talk Task

The talk mat is deliberately very busy in order to emphasise how much can be known from one fact. You can fold it to focus attention on fewer models. The chosen representation is an array of counters grouped in two different ways with the value of the counters changing.

Look at the first two arrays and discuss the calculations they can represent, reviewing that multiplication is commutative and the relationship with the corresponding division facts.

The next set of arrays has the value of 10 for each counter and discuss, explain and write calculations making connections with the previous arrays.

- *Two groups of 3 tens is 6 tens, three groups of 20 is 60*
- *60 split into groups of 30 is 2 groups, 60 shared into 2 groups is 30 per group*
- *60 split into groups of 20 is 3 groups, 60 shared into 3 groups is 20 per group*

Link the original arrays by showing the multiplication as 2×3×10.

Repeat this experience with the counter value changed to 100.

The longer arrays at the end can represent 30 × 20 and 20 × 30. Discuss, explain and write.

- *Can you see groups of 30? Can you see ten of the 30 × 2 array?* 30 × 2 × 10

Repeat with the other long array and discuss the division calculation attached to each array. Pause and look at how many calculations came from one known fact. Imagine how many more!

Activity

The activity sheet provides a similar experience and then has a variety of worded problems that can be solved with calculations derived from one fact.

Extend this task by using counters to create representations for another known multiplication fact and derive calculations. Challenge students to create a variety of worded problems for the calculations.

Video guidance

Pack 3 Session D
Activity: Derived facts

Answers

Copy and complete the calculations this array could represent as the value of each counter is changed.

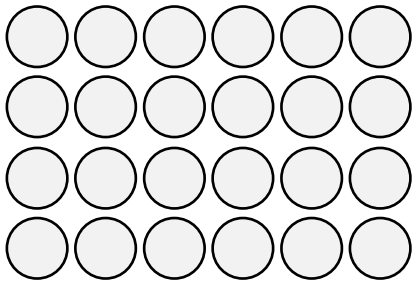
Each counter has a value of 1

$4 \times 6 = 24$

$6 \times 4 = 24$

$24 \div 6 = 4$

$24 \div 4 = 6$



Each counter has a value of 10

$40 \times 6 = 240$

$6 \times 40 = 240$

$240 \div 6 = 40$

$240 \div 40 = 6$

$4 \times 60 = 240$

$60 \times 4 = 240$

$240 \div 60 = 4$

$240 \div 4 = 60$

Use the fact that $4 \times 7 = 28$ to answer the following.

I do 40 minutes of exercise every day. How many minutes will I have done after 7 days?

$4 \times 7 = 28$
$40 \times 7 = 280$

280 grams of sugar is split into bowls with 40g in each. How many bowls of sugar are there?

$28 \div 4 = 7$
$280 \div 40 = 7$

Completing a level of a game gets you 70 points. You manage to do 40 levels, how many points do you have?

$4 \times 7 = 28$
$40 \times 70 = 2800$

£280 is shared equally between 4 people. How much does each get?

$28 \div 4 = 7$
$280 \div 4 = 70$

Mathematics Mastery

Step-by-step

Pack 4: Multiplication strategies

Session A: Adjust a factor by 1

Resources needed: Dienes thousands, hundreds, tens and ones

The purpose of this session is to explore the relationships between multiplication facts where one factor is one more or one less. You want to help pupils realise that they can use known facts to work out other facts

Talk Task
A situation is presented with seven bags each containing eight apples. Ask pupils to explain why the given calculation describes the total number of apples.

Think about what happens when you adjust the situation and discuss and write calculations that can show the result each time. Think about the relationships between the calculations explored so far.

E.g. *9 groups of 7 is 7 more than 8 groups of 7*

The second set of calculations are outside of the 'times tables' in order to prompt pupils to use the given fact rather than work out each calculation separately. Although, it is also useful to work each one out to check.

A calculation is placed on a number line and pupils are prompted to use this to place related facts on the line. Think about the relationship between the calculations by completing the sentences.

The parts of a multiplication calculation can be called **factor × factor = product**

This can be useful language when trying to describe the general relationships explored.

If a factor is decreased by 1, the product is decreased by the value of the other factor.

If a factor is increased by 1, the product is increased by the value of the other factor.

Activity
The activity sheet guides students through tasks similar to the talk task. Then chains of calculations are used to help pupils think about the relationship between multiples of 5 and multiples of 6 and between multiples of 9 and multiples of 10.

Extend the activity by asking students to select a calculation and write related multiplication and division facts. Challenge them to make or draw models to represent the calculations.

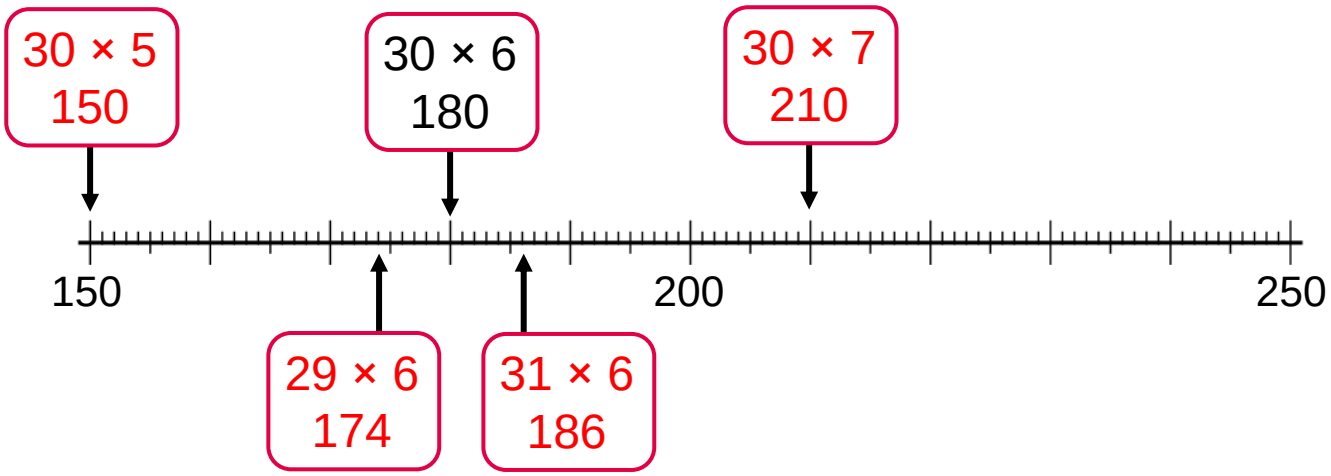
Video guidance

Pack 4 Session A

Answers

Activity: Derived facts – adjusting a factor by 1

1) Use the known fact to place the calculations onto the number line and complete the statements to describe the relationship.



29 × 6 is 6 less than 30 × 6

31 × 6 is 6 more than 30 × 6

30 × 5 is 30 less than 30 × 6

30 × 7 is 30 more than 30 × 6

2) Complete the calculations. What relationships do you notice..

$3 \times 5 + 3 = 3 \times 6$

$4 \times 5 + 4 = 4 \times 6$

$5 \times 5 + 5 = 5 \times 6$

$6 \times 5 + 6 = 6 \times 6$

$7 \times 5 + 7 = 7 \times 6$

$9 \times 2 = 20 - 2$

$9 \times 3 = 30 - 3$

$9 \times 4 = 40 - 4$

$9 \times 5 = 50 - 5$

$9 \times 6 = 60 - 6$

$9 \times 14 = 140 - 14$

Mathematics Mastery

Step-by-step

Pack 4: Multiplication strategies

Session B: Monthly payments

Resources needed: Individual whiteboard to draw number lines

The purpose of this session is to use what they know about related multiplication facts to multiply a 2-digit number. A table of values and a double number line are used to record information and a variety of strategies should be explored.

Talk Task

Connect the context of the talk task to students' experiences by asking if they have ever had pocket money, or a job that paid the same amount on a regular basis. Do they (or do they know anyone) who makes regular payments each month?

Discuss the first situation and the table of values, asking students to explain how the table relates to the situation.

The table shows how much it will cost after 1 month, 2 months and 10 months.

Explore strategies to find other values in the table

5 months will be half the cost of 10 months. Half of 18 is 9. Half of 180 is 90.

6 months will be £18 more than 5 months.

4 months is double the cost of 2 months or £18 less than the cost of 5 months.

A double number line is a useful tool for seeing relationships and keeping track of calculation steps. Explore the information on the number line, connect to the table and the context and record more values.

Repeat a similar experience with the second situation.

Extend the activity by asking students to extend the double number lines.

What values can you record further along the line? How much will it cost for 2 years, 24 months?

Activity

The activity sheet guides students through using tables and double number lines in the context of monthly payments across a year. The last situation does not give the value of one payment. Support might be needed to think about how to use the information given.

The difference between 5 and 6 months will be the cost of a month. I can use 5 months to find the cost of 10 months.

Video guidance

Pack 4 Session B

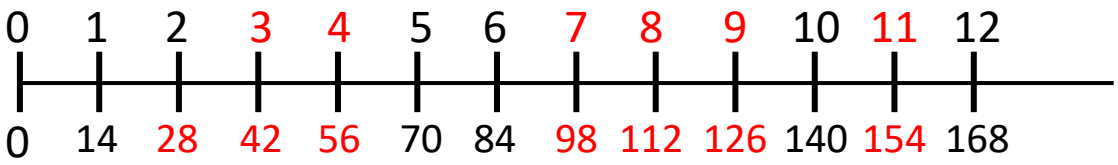
Answers

Activity: Monthly payments

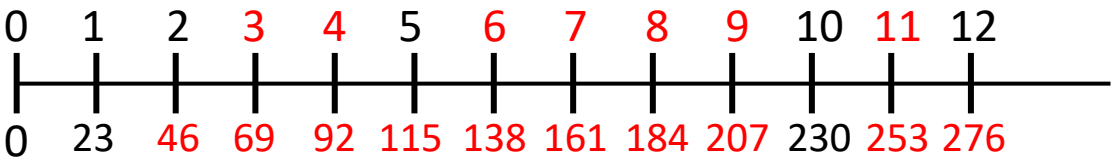
For each situation, write as much information as you can about the cost across a year.

My contact lenses cost £14 each month.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Cost	14	28	42	56	70	84	98	112	126	140	154	168

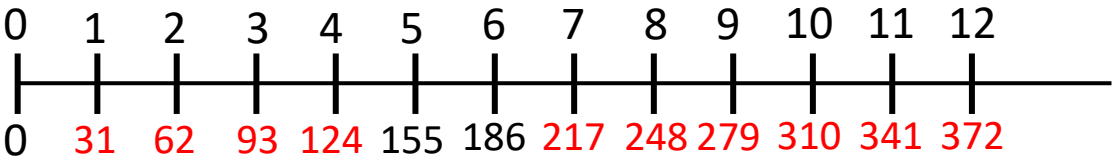


My mobile phone costs £23 each month.



After 5 months I have paid £155

After 6 months I have paid £186



Mathematics Mastery

Step-by-step

Pack 4: Multiplication strategies

Session C: Adjusting a factor by 10

Resources needed: Dienes hundreds, tens and ones

The purpose of this session is to explore partitioning strategies and the relationship between calculations where a factor is ten more or less. Dienes and area models are used to prompt discussion and reveal the structure.

Talk Task
Ask pupils to describe the Dienes and area models on the talk mat.

What do you see? What do you notice?

Take the time to discuss and explain how each model shows each calculation, discussing what has changed as you move down the page.

From the second model, the calculations are written in two ways. Ask pupils to explain how the model shows the calculation.

The two digit number is partitioned into tens and ones and each part is multiplied. This is shown in the tens and ones.

Continue to discuss the models and calculations, working out the answer and explaining why the calculations are equal.

At the end of the page, a double number line is shown with the values from the model above. Spend time describing how the number line is connected to the Dienes and the calculations. If you have not already done so, focus on the difference between the total value for each model/calculation and discuss why it increases by 40.

Extend the activity by asking students to imagine the sequence of calculations continuing. *What would come next? What would the model look like?*

Activity
The activity sheet provides a similar experience as in the talk task connecting a model of Dienes to area models using rectangles.

Students have the opportunity to sketch models to represent calculations and complete statements to describe the relationship between calculations.

Extend the activity by building models with Dienes and sketching area models and writing abstract calculations..

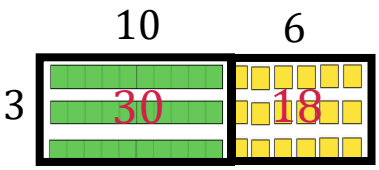
Video guidance

Answers

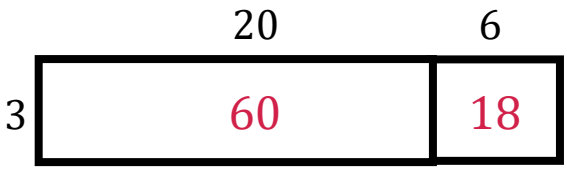
Pack 4 Session C

Activity: Derived facts –adjusting by a factor by 10

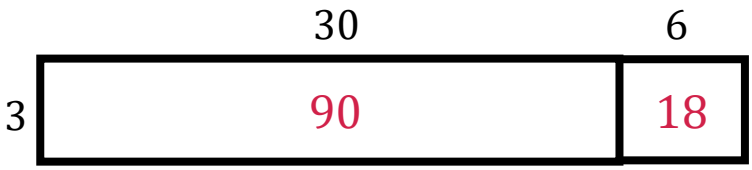
1) Label the area models and complete the calculations.



$16 \times 3 = \mathbf{30} + 18 = \mathbf{48}$



$26 \times 3 = \mathbf{60} + 18 = \mathbf{78}$

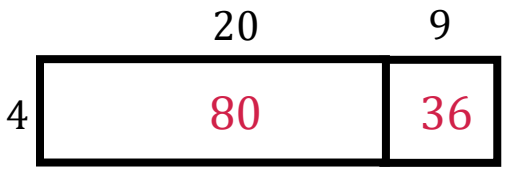


$36 \times 3 = \mathbf{90} + 18 = \mathbf{108}$

2) Draw models to represent multiplication calculations

Draw an array with Dienes to represent 24 × 3

Draw and label a rectangle to represent 29 × 4



3) Complete the statements.

14 × 5 is 50 more than **4** × 5

8 × 3 is 30 less than 18 × 3

16 × **4** is 40 more than 6 × 4

8 × 7 is 70 less than **18** × 7

Mathematics Mastery

Step-by-step

Pack 4: Multiplication strategies

Session D: Exploring calculation strategies

Resources needed: Dienes thousands, hundreds, tens and ones

The purpose of this session is to explore different ways to complete the same calculation and describe how strategies works. Understanding form previous sessions and packs is drawn upon to develop flexibility when calculating.

Talk Task

The talk mat show four different strategies for calculating 75 × 4. The answer, 300, is provided as the purpose is not to find the answer but instead to explore the different strategies and explain how they work.

Each strategy has been represented with a model. Take the time to think about each model and describe how each step of the calculation strategy is shown. Below are some suggestions for ways to talk about each model:

The bars double in length as you go down. Doubling and doubling again is the same as multiplying by 4.

The open array shows that 75 has been split into 70 and 5 and each part multiplied by 4.

This open array shows 80 × 4 is 20 more than 75 × 4

The counters show that 75 is split into 3 lots of 25. There are 12 lots of 25 and 4 × 25 is 100.

Discuss which strategy is the most efficient. There is no definite answer to this and the purpose of the discussion is not to decide which but rather to think about what makes a strategy efficient.

Activity

The activity sheet has two models representing different strategies for calculating 15 × 8 and students are to complete the steps of the calculations. Then students are given space to record three different strategies for completing the same calculation. Encourage them to draw and write for each.

Extend the activity by discussing names for different strategies you have used and thinking of other calculations when you would use each. For example, a doubling strategy can be used when multiplying by four. Draw similar models to represent other calculations.

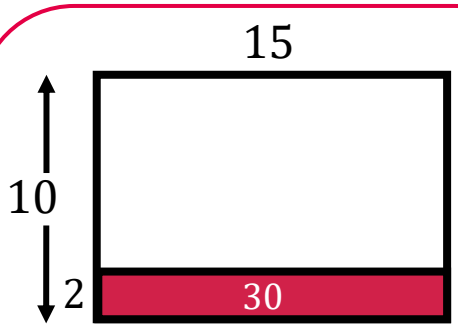
Video guidance

Pack 4 Session D

Activity: Exploring calculation strategies

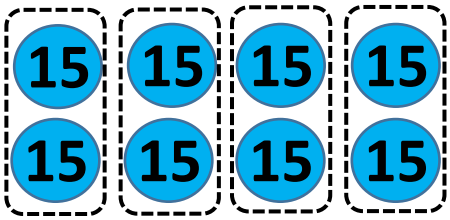
Answers

1) Complete the calculations for two ways to calculate 15 × 8



$15 \times 8 = 15 \times 10 - 15 \times \mathbf{2}$

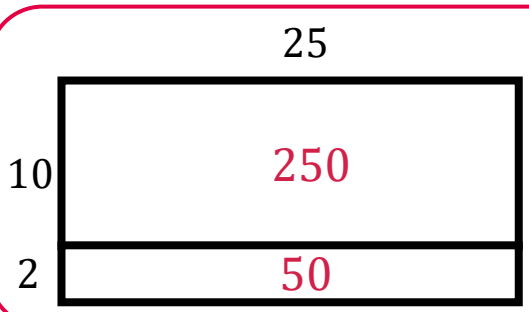
$= \mathbf{150} - 30$



$15 \times 8 = 15 \times 2 \times \mathbf{4}$

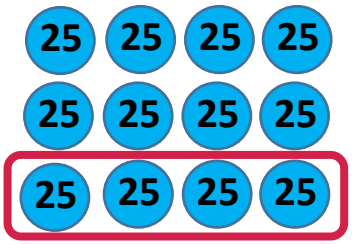
$= \mathbf{30} \times 4$

2) Show with models and calculations three different ways to calculate 25 × 12



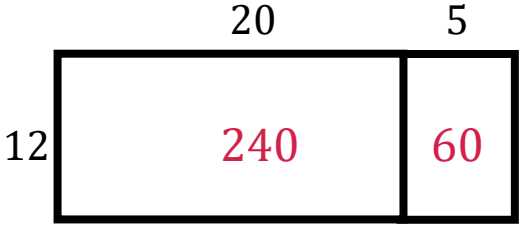
$25 \times 12 = 25 \times 10 + 25 \times 2$

$= 250 + 50$



$25 \times 12 = 25 \times 4 \times 3$

$= 100 \times 3$



$25 \times 12 = 20 \times 12 + 5 \times 12$

$= 240 + 60$

Mathematics Mastery

Step-by-step

Pack 11: Division

Session A: Division and multiplication

Resources needed: Counters or other countable materials

The purpose of this session is to revise understanding of the connection between multiplication and division and use knowledge of multiples to make sense of division calculations.

Talk Task

An array of counters each labelled with the value of 10 is presented. Ask pupils to tell you what they can see and describe any calculations they think the model represents. This should be a familiar model of multiplication and pupils should be able to identify and describe equal groups and write multiplication calculations.

Use the suggested sentences to discuss the language 'multiple of' and 'divisible by'. Connect these ideas using the array of counters to support explanations.

- 210 is a multiple of 70
- 210 is divisible by 70. I can divide 210 into 3 groups of 70
- 210 is a multiple of 3
- 210 is divisible by 3. I can divide 210 into 3 equal groups. Each group is 70.

Think carefully about how to describe the division based on the model. For example, it is not easy to see 210 divided into 70 groups of 3 with this model.

- 210 is a multiple of 30
- 210 is divisible by 30. I can divide 210 into 7 groups of 30.
- 210 is a multiple of 7
- 210 is divisible by 7. I can divide 210 into 7 equal groups. Each group is 30.

While working through these, write division calculations that the array can represent. Pause and reflect on the many calculations that can be worked out.

For the next section, build on the derived facts identified so far to identify numbers that are divisible by 7. A number line is provided that restricts the task to numbers between 130 and 230. For each number identified, discuss how you know and how to record as a division calculation and a multiplication calculation.

Activity
This activity guides students through a similar experience of deriving and recording facts. Then a multiplication facts is given and pupils are to use this to solve word problems involving division and related facts. Extend this task by asking pupils to generate their own word problems.

Video guidance

Pack 11 Session A

Activity: Division and multiplication

Answers

1) Copy and complete the calculations this array could represent as the value of each counter is changed.

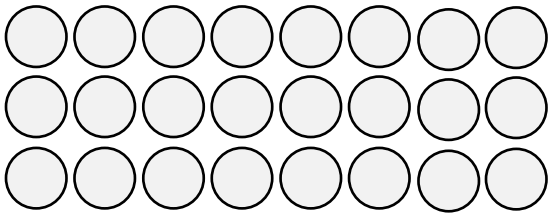
a) Each counter has a value of (1)

$3 \times 8 = 24$

$8 \times 3 = 24$

$24 \div 8 = 3$

$24 \div 3 = 8$



b) Each counter has a value of (10)

$30 \times 8 = 240$

$8 \times 30 = 240$

$240 \div 8 = 30$

$240 \div 30 = 8$

$3 \times 80 = 240$

$80 \times 3 = 240$

$240 \div 3 = 80$

$240 \div 80 = 3$

$24 \times 10 = 240$

$10 \times 24 = 240$

$240 \div 10 = 24$

$240 \div 24 = 10$

2) Use the fact that $4 \times 6 = 24$ to answer the following:

£240 is shared equally between 4 people. How much does each person get? £60	240 grams of sugar is split into bowls with 60 g in each. How many bowls of sugar are there? 4 bowls of sugar
Completing a level of a game gets you 60 points. You have 2400 points. How many levels have you completed? 40 levels	I do 40 minutes of exercise every day. How many days until I have done 240 minutes? 6 days

Mathematics Mastery

Step-by-step

Pack 11: Division

Session B: Halving strategies

Resources needed: Dienes

The purpose of this session is to explore different division strategies that involve halving. Exploring different ways to complete the same calculation allows you to discuss which strategy you would choose.

Talk Task

Four models are shown to represent different ways of seeing half of 72. Start by asking pupils to describe what they can see and discuss what is the same and what is different. Encourage pupils to build and label each model.

Below the models are four sets of calculations describing the steps of four different strategies. Read and discuss which matches which model and think about why and how you can see each calculation in the arrangement of the blocks. Take the time to attach each step of the calculation to the model that it matches, describing the role of each number.

The top two strategies partition 72 and halve each part. On the left, 72 is partitioned into 70 and 2 and on the right into 60 and 12. It is importance to encourage pupils to be flexible in their choices of how to partition and consider options other than tens and ones.

The bottom two strategies identify a multiplication calculation and halve one of the factors to halve the product.

Having discussed each model and strategy, talk about which you would choose to use for this calculation and think about other calculations that suit each strategy. Extend this task by exploring a similar calculation such as $58 \div 2$ or $96 \div 2$ and building or drawing models that show different ways this could be calculated. The focus should be on explaining and seeing the structure.

Activity

The activity sheet uses arrays of counters to illustrate division strategies involving halving and repeated halving. Pupils are to look at each step and complete the model by drawing rings around sections of the array and complete the empty boxes

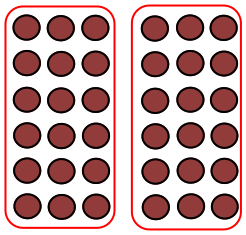
Extend this task by challenging students to find other calculations that suit each of the strategies explored as well as calculation that do not.

Video guidance

Pack 11 Session B
Activity: Halving strategies

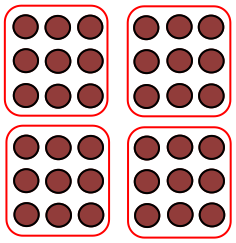
Answers

1) The images show a halving strategy. Complete the boxes.



Two groups of 18

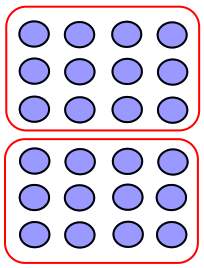
$36 \div 2 = 18$



Four groups of 9

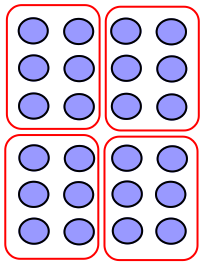
$36 \div 4 = 9$

2) Complete the images to match the steps of the halving strategy.



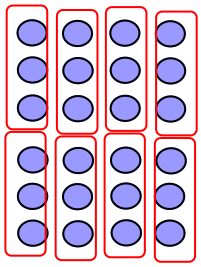
Half of 24 is 12

$24 \div 2 = 12$



Half of 12 is 6

$24 \div 4 = 6$



Half of 6 is 3

$24 \div 8 = 3$

3) Complete the strategy and show it works with another calculation.

To divide a number by 6, I can halve and then divide by 3

Half of 48 is 24

24 divide by 3 is 8

$48 \div 6 = 8$

Half of 186 is 96

96 divide by 3 is 32

$186 \div 6 = 32$

There are different ways to complete question 2 and many different examples that could be given for question 3.

Mathematics Mastery

Step-by-step

Pack 11: Division

Session C: Division structures

Resources needed: A way to draw number lines

The purpose of this session is to explore different ways of interpreting the same division calculation. Division as sharing and division as grouping. For example, $10 \div 2$ can mean 10 shared between 2 or it can mean 10 put into groups of 2. Number lines are used as the model for exploring this.

Talk Task

Division calculations can be interpreted in two different ways and these different ways of seeing division can lead to different calculation strategies. This task uses number lines as the model to explore the same calculation in two different situations.

Read what each person says and think about what is the same and what is different between the two situations.

- *In both we know the total number and it is the same,* 150
- *We know the number of pencils in each pack,* 30*. We don't know the number of packs.*
- *We don't know the number of pens in each pack. We know the number of packs,* 30*.*

Ask pupils to explain why the same calculation, $150 \div 30$ can be used to solve both situations. Highlight that this calculation can mean 'how many 30s in 150?' or it can mean '150 is split into 30 equal groups, how many in each group?'

Look at the number line for each situation and spend time making sense of the information displayed. Ask pupils to place more information onto each line.

One pack is 30 *pencils, how many packs of* 30 *is* 150*? Two packs is* 60*, four packs is* 120 *and five packs is* 150 *pencils.*

150 *pens is the amount in* 30 *packs, what is* 150 *divided into* 30 *equal parts?* 10 *packs is* 150 *divided into* 3 *which is* 50*. If* 10 *packs is* 50 *then* 1 *pack has* 5 *pens.*

Activity

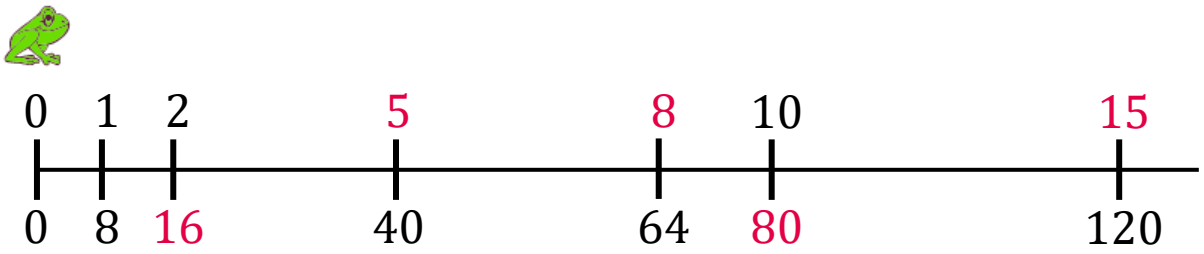
The activity sheet uses the context of frogs jumping to provide further practice with using a number line. Extend the activity by asking pupils to record multiplication and division calculations that each question is connected with. They can also generate their own situations to show the two interpretations of division.

Video guidance

Pack 11 Session C
Activity: Division structures

Answers

1) A frog travels 8 cm for each jump.



a) How far has it travelled after 2 jumps? 16 cm

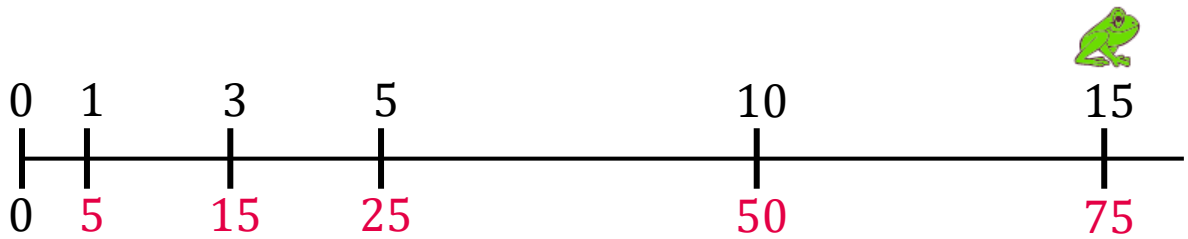
b) How many jumps does it take to travel 40 cm? 5

c) How many jumps does it take to travel 64 cm? 8

d) How far has it travelled after 10 jumps? 80 cm

e) How many jumps does it take to travel 120 cm? 15

2) This frog has jumped 15 equal jumps and travelled 75 cm.



a) How far how it travelled after 5 jumps? 25 cm

b) How far has it travelled after 10 jumps? 50 cm

c) How big is each jump? 5 cm

d) How far has it travelled after 3 jumps? 15 cm

Mathematics Mastery

Step-by-step

Pack 11: Division

Session D: Division with remainders

Resources needed: Dienes

The purpose of this session is to explore different models for representing division. All of the models have been used in previous packs to represent multiplication showing that knowledge of multiples can be used to divide.

Talk Task

Three different models are provided with some labels. Start by asking pupils to describe what they see, encouraging them to label the models and think about calculations they could represent.

Three division calculations are given. Take time to match each calculation to a representations prompting pupils to give clear explanations and show why.

The array of Dienes shows 9 tens and 3 ones which is 93. This has been divided into 3 equal rows and each row has length 31. This can represent the multiplication calculation $31 \times 3 = 93$ and so it also represents the division calculation $93 \div 3 = 31$.

The rectangle is labelled with an area of 124. One side has length 4 and this information connects it to the division calculation $124 \div 4$. Take time to discuss why the other length must be 30 and 1.

The number line shows one part with a value of 3, 30 parts with a value of 90, and asks what number of parts have a value of 96. It is asking how many threes are there in 96. This is the division calculation $96 \div 3 = 32$

Having discussed each of the models and the calculations they represent, discuss what is the same, what is different and what is the relationship between these calculations. You can extend the session by creating the other models for each calculation and by varying the calculations in other ways and representing those.

Activity

The activity sheet provides similar experiences of completing calculations, labelling models and using understanding of multiplication to divide. The final question asks pupils to create a representation for a given calculation. There are lots of possible ways to do this and you can encourage pupils to create more than one representation.

Video guidance

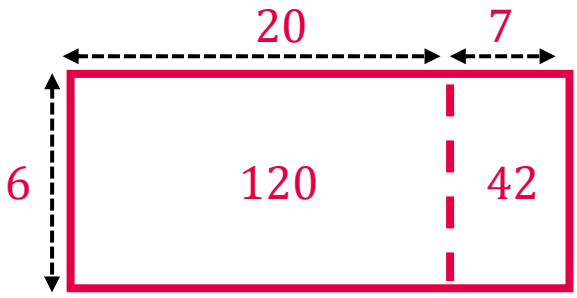
Pack 11 Session D
Activity: Models of division

Answers

1) Label the models and complete the calculations.

$92 \div 4 = 23$

$23 \times 4 = 92$



$162 \div 6 = 27$

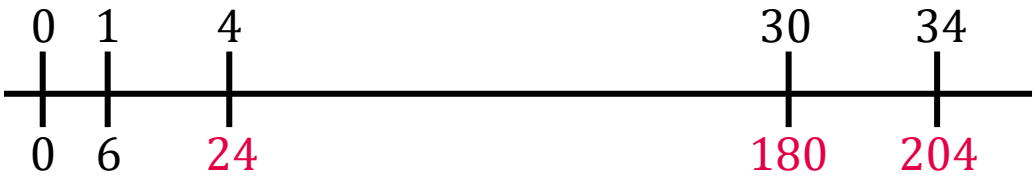
$27 \times 4 = 162$

2) Complete the calculations and label the number line.

a) $4 \times 6 = 24$ $\quad 24 \div 6 = 4$

b) $30 \times 6 = 180$ $\quad 180 \div 6 = 30$

c) $34 \times 6 = 204$ $\quad 204 \div 6 = 34$



3) Draw a model to represent $72 \div 3 = 23$.

There are lots of ways to complete this. Look for models that show 72 as 3 groups of 23 or as 23 groups of 3.

Mathematics Mastery

Loved a session?

Got some ideas for improvements?

Spotted a typo?

Let us know your feedback here