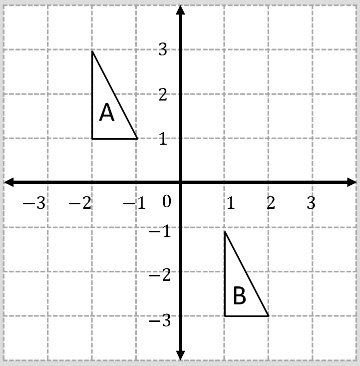
Mathematics Mastery
Maths Pack

Exercises

Week 1: Transformations

Session 1: Translation

Concept Corner

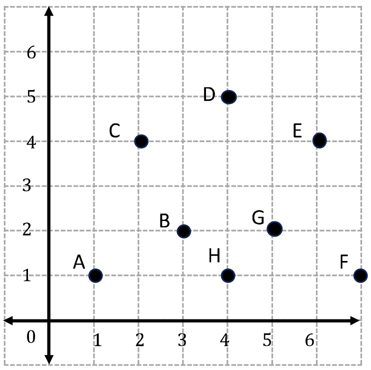


3 | translated | 4 | direction | $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ | vector

We say a shape has been ____________ when every point in the shape moves by the same distance in the same ______________. We can use a ________ to describe the translation.

e.g. The transformation from A to B is a translation by the vector _____. The shape moves _______ to the right and _______ down.

1.



a) Write down the coordinates of the points A – H

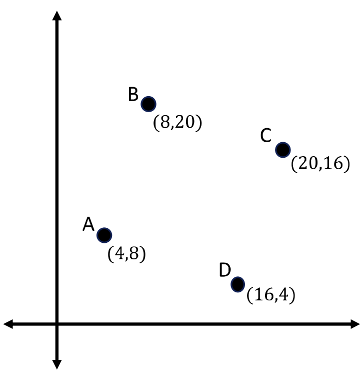
b) Describe the journey using vector notation:

e.g. A to B: $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

i) C to D ii) D to E iii) F to A

iv) D to C v) E to D vi) A to F

2. Describe the journey using vector notation:

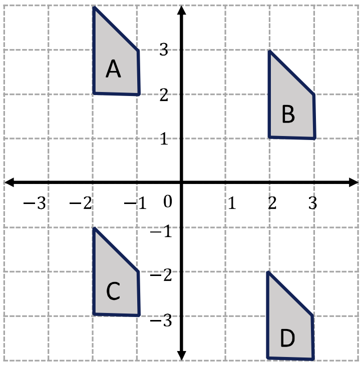


a) A to B b) D to C c) D to A

d) C to B e) D to B f) A to C

Mathematics Mastery Ark

3. Describe the transformation in each case:



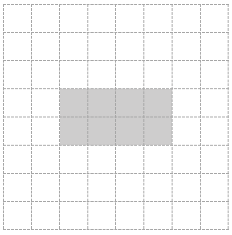
a) A to B b) B to D c) A to D

d) A to C e) C to D f) D to A

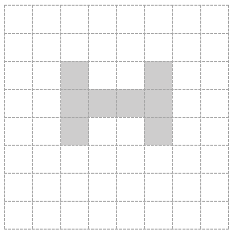
4. Sketch, on a single diagram, the outcome of translating the shape by:

$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$, $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

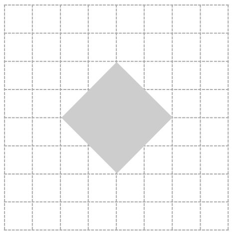
a)



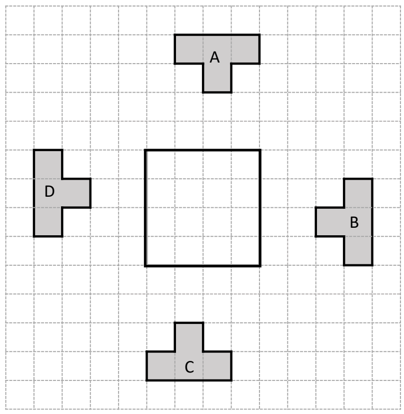
b)



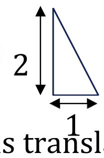
c)

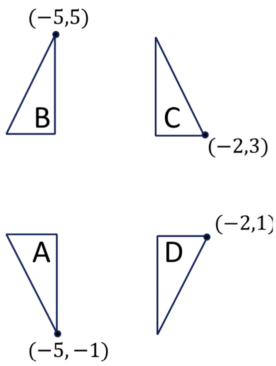


5. Describe four translations of the octagons that sends the shapes inside the square forming a tessellation pattern.



Questions for depth:

1. Four copies of the triangle  are arranged as follows:

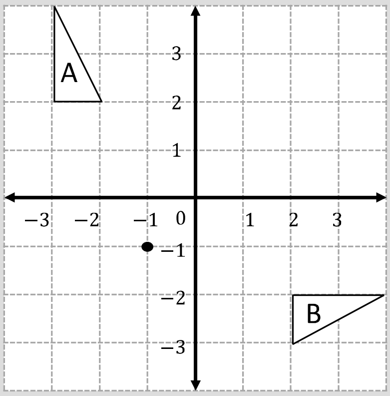


a) If A is translated by $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, B is translated by $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and C by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ what quadrilateral is formed?

b) A is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, B is translated by $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and C by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ what quadrilateral is formed?

Week 1 Session 2: Rotation

Concept Corner



(−1,1)	rotate
90°	rotation
90°	anticlockwise

We can ___________a shape about a point called the centre of ___________ .

e.g. The transformation from A to B can be described as a rotation of _______ clockwise about the point ______ . The transformation from B to A can be described as a rotation of ______ _______________ about the same point.

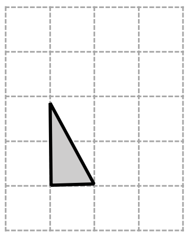
1. Draw a copy of each shape after:

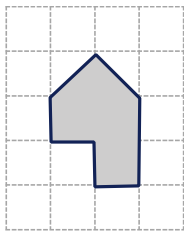
a) a 90° rotation clockwise

b) a 180° rotation

c) a 270° rotation clockwise

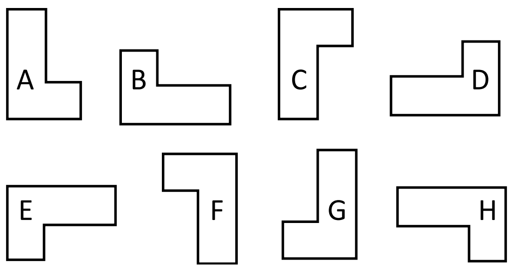
d) a 90° rotation anticlockwise





2. Generate **five statements** describing the angle and direction of rotation between two shapes:

   e.g. “H to C is a rotation of 270° clockwise.”

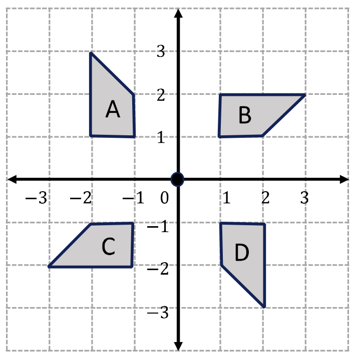


3. Describe the following transformations:

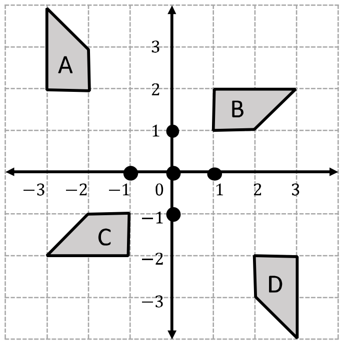
E.g. C to A “a rotation of 90° clockwise about the origin.”

a) A to B　　b) B to D　　c) A to D

d) A to C　　e) C to D　　f) D to A



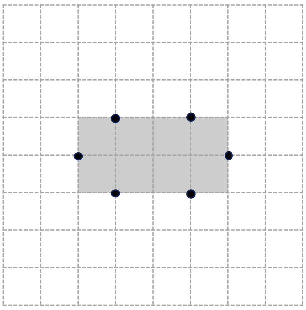
4. Describe the following transformations:

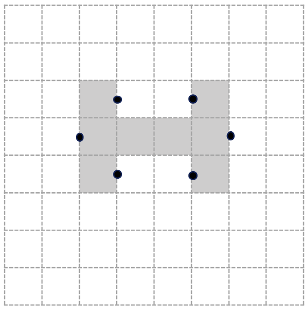


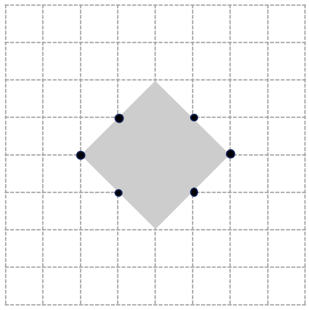
a) A to B b) B to D c) A to D

d) A to C e) C to D f) D to A

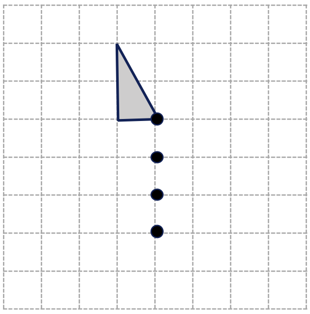
5. Sketch, on a single diagram, the outcome of rotating the shape by 180° about each of the six points:

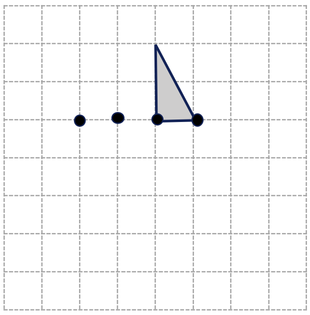
a) 

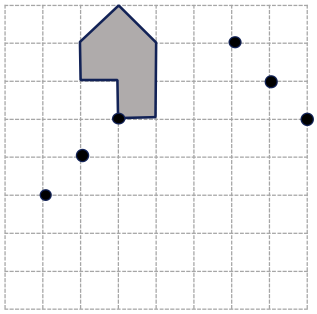
b) 

c) 

6. Sketch, on a single diagram, the outcome of rotating the shape by 90° clockwise about each of the points:

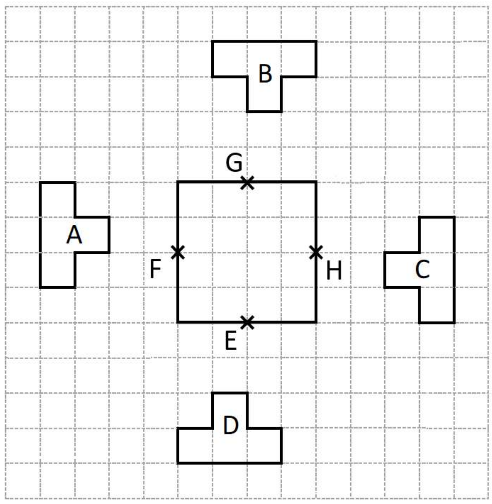
a) 

b) 

c) 

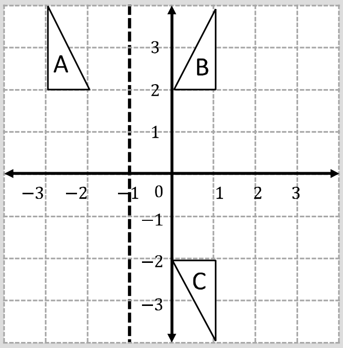
Questions for depth:

1. Describe four rotations of the octagons that sends the shapes inside the square forming a tessellation pattern.



Week 1 Session 3: Reflection

Concept Corner

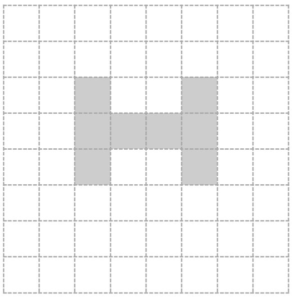
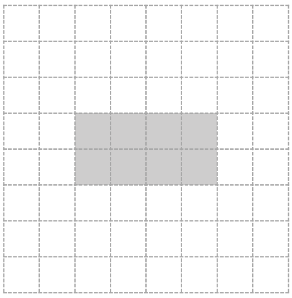
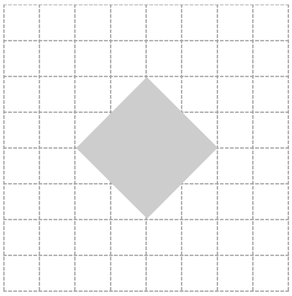


$x = -1$	reflect
equidistant	
$y = 0$	line

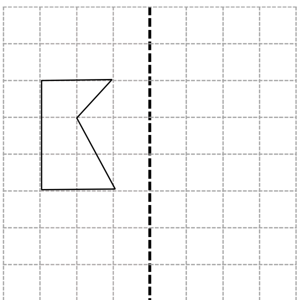
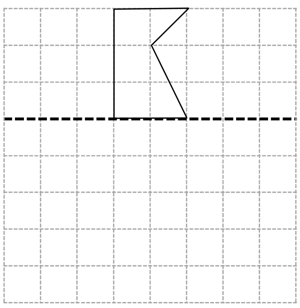
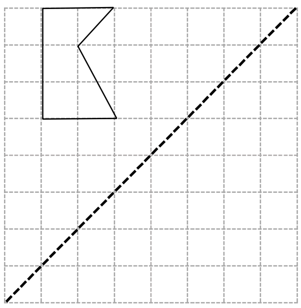
We can ____________a shape in a __________. A point and it's reflection are ____________ from the line of reflection.

e.g. The transformation from A to B can be described as a reflection in the line _____.
The transformation from B to C can be described as a reflection in the line _____.

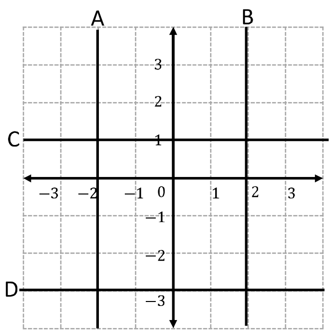
1. Identify the number of lines of symmetry :

a) 
b) 
c) 

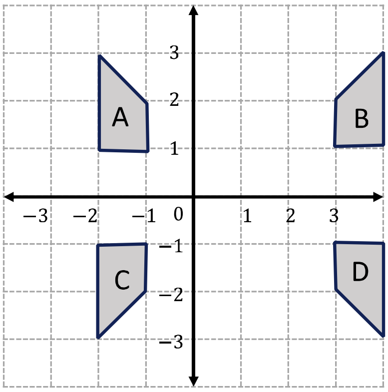
2. Copy the shape and reflect it in the dotted line.

a) 
b) 
c) 

3. Write the equation for each line:



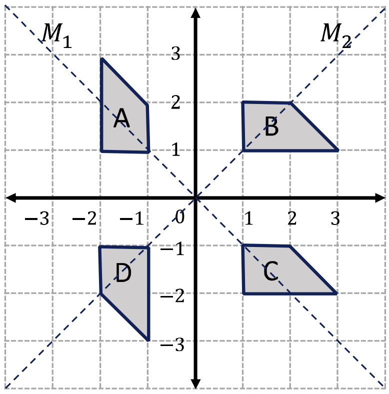
4. Describe the following transformations:



a) A to B b) B to D c) A to D

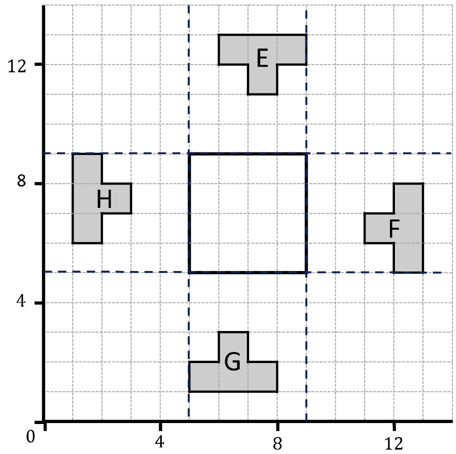
d) A to C e) C to D f) D to A

5. Two lines M_1 and M_2 are shown. Describe the following transformations:

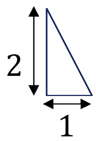


i) B to D ii) D to B iii) A to C

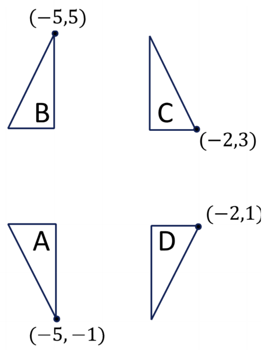
6. Describe four reflections of the octagons that sends the shapes inside the square forming a tessellation pattern.



Questions for depth:

1. Four copies of the triangle 

are arranged as follows:



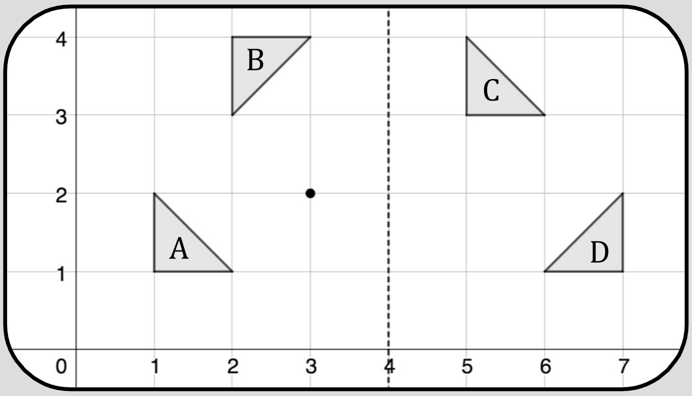
a) Describe the transformation from B to C

b) Describe the transformation from A to D

c) Describe the transformation from A to C

Week 1 Session 4: Isometries

Concept Corner



4	B	rotated
C	translated	
D	reflected	

Reflection, rotation and translation are all examples of isometries.

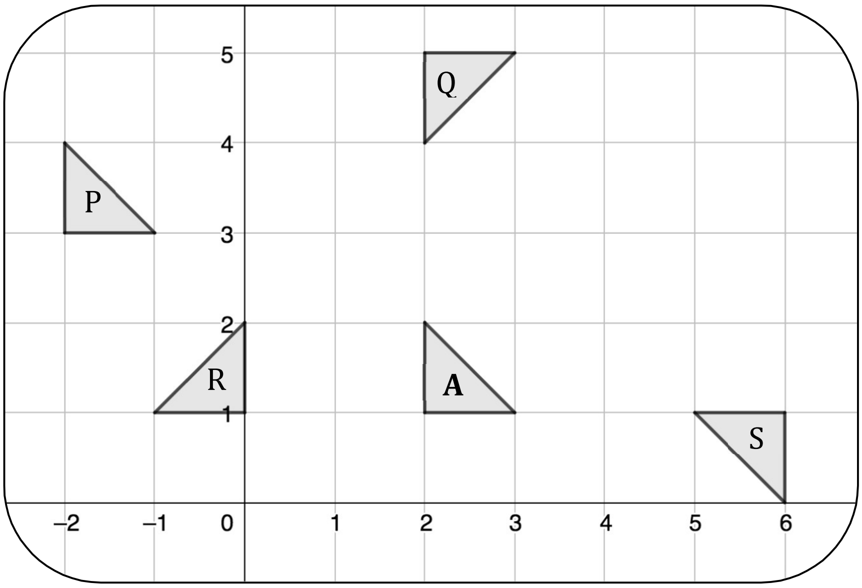
Triangle A has been ____________ 90° clockwise about (3, 2) to give triangle ____

Triangle A has been ____________ in the line $x =$ ____ to give triangle ____

Triangle A has been ____________ by $\binom{-4}{-2}$ to give triangle ____

1. Look at the image below. Write down **single transformations** to which could transform:

a) A onto R
b) A onto P
c) A onto Q
d) A onto S

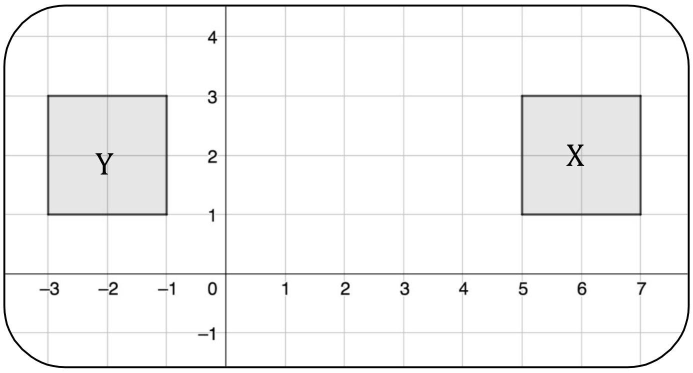


2. Look at the triangles in Q1.
a) What single transformation could transform Q onto S?
b) What single transformation could transform R onto Q?

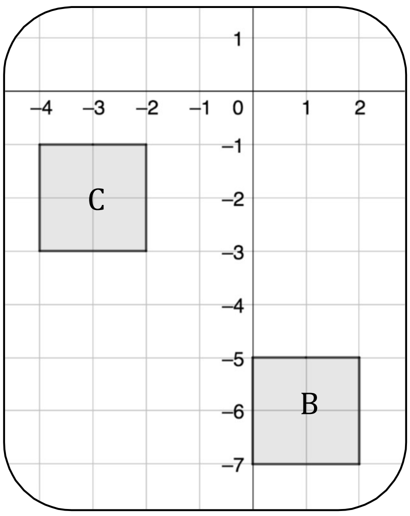
3. In your book draw a set of axes where both x- and y-axes range from 0-10.
Copy triangle A from Q1 onto your axes.
Now, on your axes, draw the images created by the following transformations:

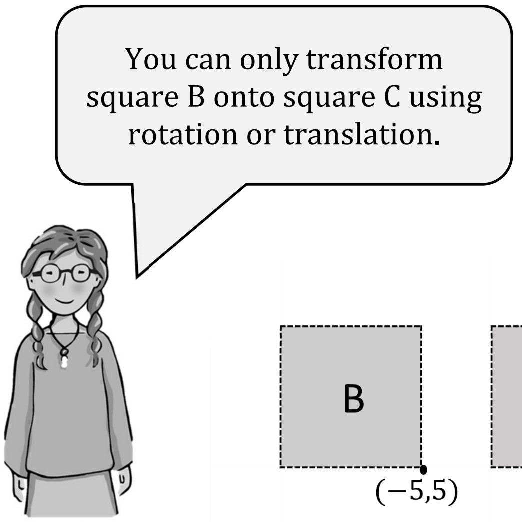
a) Translate A by $\binom{5}{-1}$
b) Reflect A in the line $y = 5$
c) Rotate A 90° clockwise about $(7, 1)$

4. Look at the diagram below. Write down **three different** transformations that could have transformed square X onto square Y.



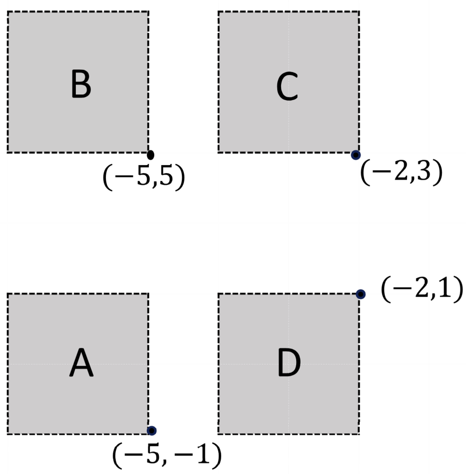
5. Look at the image below. Do you agree with Rosie's statement? Explain your answer.





Questions for depth:

1. Explore the different transformations that takes one square to another in the following image:



Mathematics Mastery
Ark

Week 2: More transformations

Session 1: Combining reflections

Concept Corner

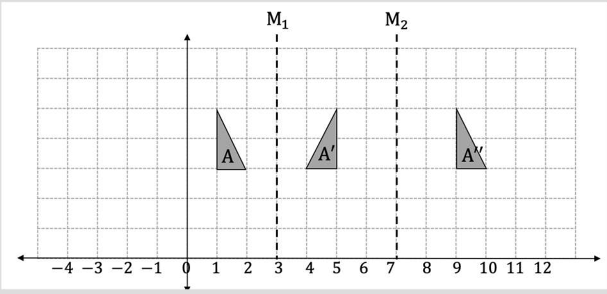
A combination of ______________ can sometimes be described by a different single transformation.

e.g. Triangle A is reflected in the line M_1 to give ___.

A' is then __________ in the line M_2 to give ____.

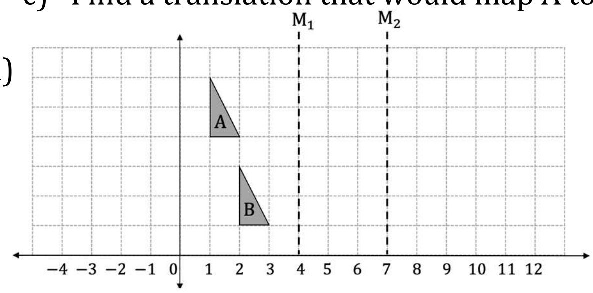
A could also be mapped to A'' by a translation by the vector _____.

transformations | $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$ | reflected | A' | A''

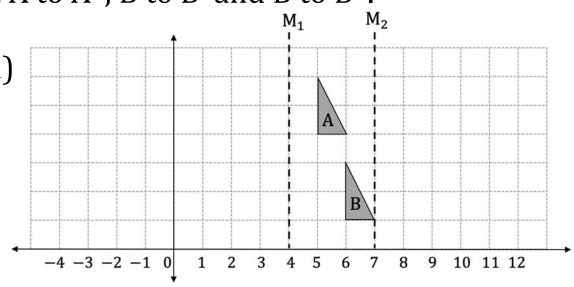


1. For each question:
   a) Reflect A and B in line M_1 followed by M_2. Label the results A' and B'.
   b) Reflect A and B in line M_2 followed by M_1. Label the results A'' and B''.
   c) Find a translation that would map A to A', A to A'', B to B' and B to B''.

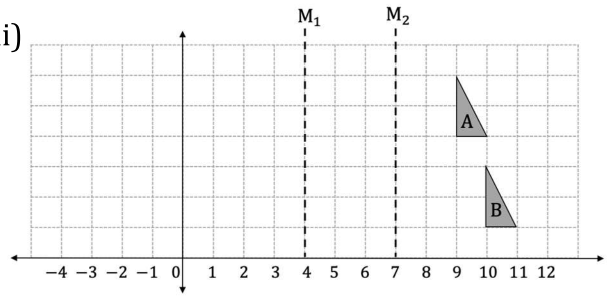
i)



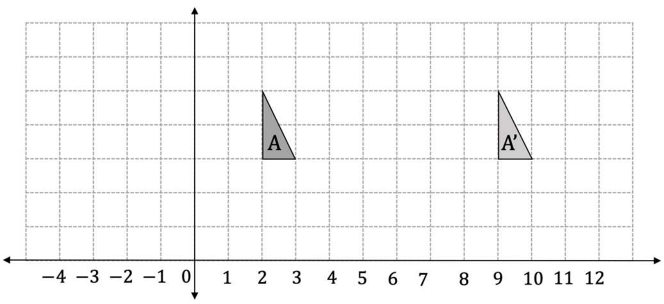
ii)



ii)

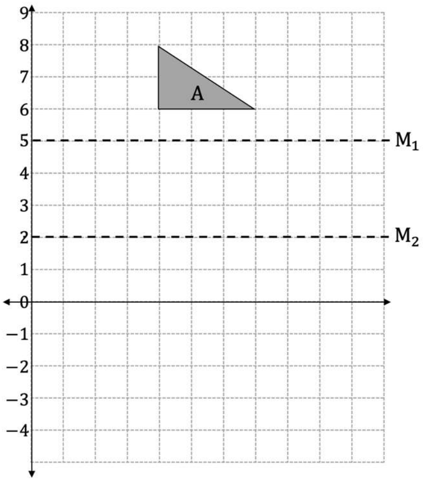


2. a) Give the translation that maps A to A'?
   b) Find the equation of two possible mirror lines that would reflect triangle A to A'.
   c) Find another three examples of two mirror lines that would map A to A'. What do all the pairs have in common?

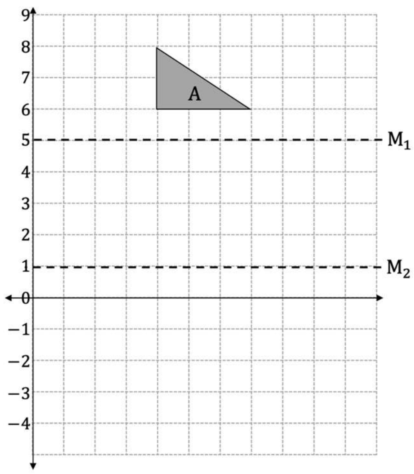


3. For each question:
   a) Reflect A line M_1 followed by M_2. Label the results A'.
   b) Find a translation that would map A to A'.
   c) Describe the relationship between this translation and the lines M_1 and M_2.

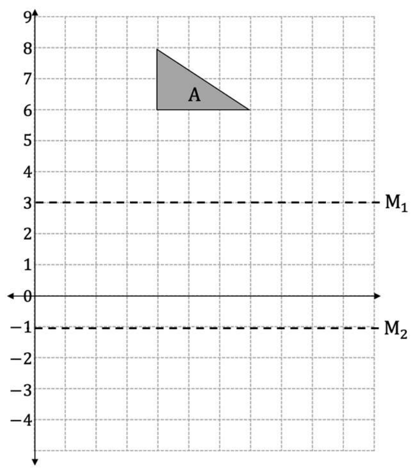
i)



ii)



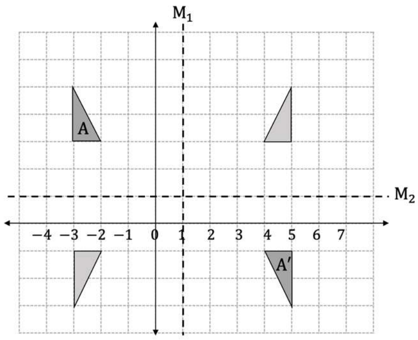
iii)

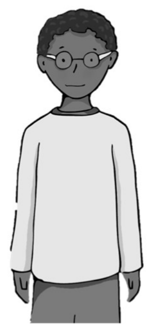


4. Sam tries reflecting the triangle A in a vertical and then a horizontal line. He notices that if he reflects in these lines in either order, the triangle ends up in the same position.

How is this different to when the two lines are parallel?

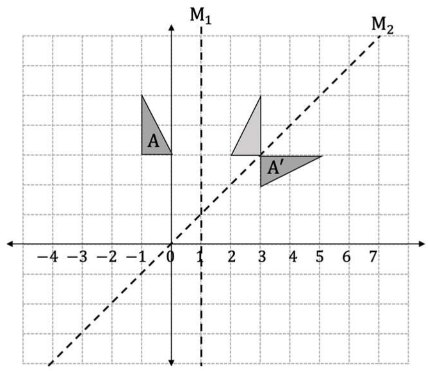
Does this always work if the two lines of reflection are perpendicular?





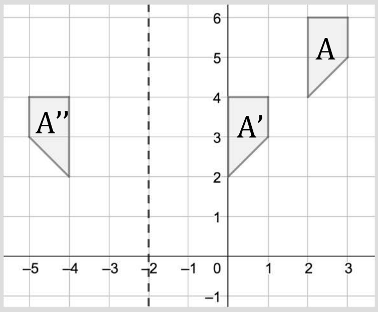
Questions for depth:

1.
   a) Reflect A line M_1 followed by M_2. Label the results
   b) What transformation would map A to A'
   c) Try moving A to a different position. What do you notice?
   d) How does this transformation relate to the lines M_1 and M_2.



Week 2 Session 2: Combining translations and reflections

Concept Corner



transformation

$\begin{pmatrix}-2\\-2\end{pmatrix}$

reflected

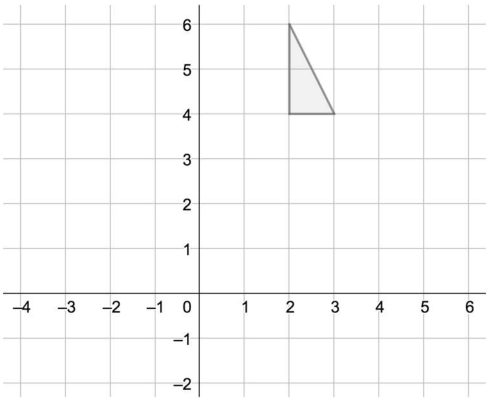
$x = -2$

We can combine a translation and a reflection by performing one ___________ after the other. For example: A is translated by the vector ____ to give A’. A’ is then __________ in the line ______ to give A’’.

1.

a) What are the co-ordinates of the vertices of this triangle?

b) State the co-ordinates of the vertices of the translated shape:



i) A translation by the vector $\begin{pmatrix}2\\0\end{pmatrix}$

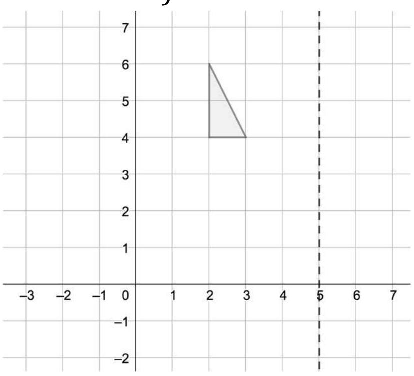
ii) A translation by the vector $\begin{pmatrix}0\\-3\end{pmatrix}$

iii) A translation by the vector $\begin{pmatrix}1\\1\end{pmatrix}$

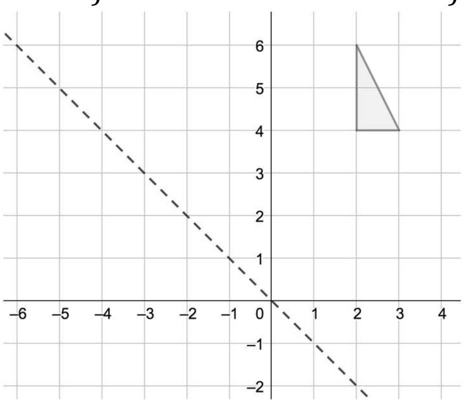
iv) A translation by the vector $\begin{pmatrix}-2\\2\end{pmatrix}$

2. State the co-ordinates of the reflected triangle:

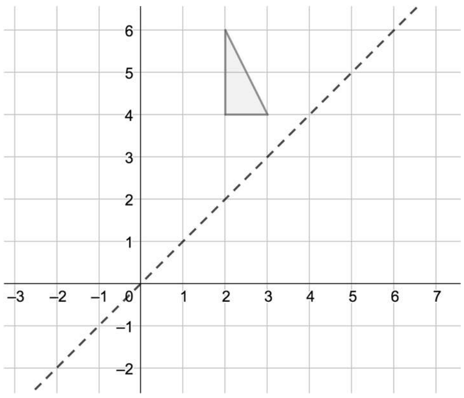
a)



b)

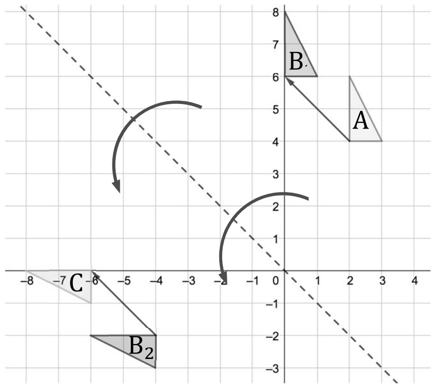


c)



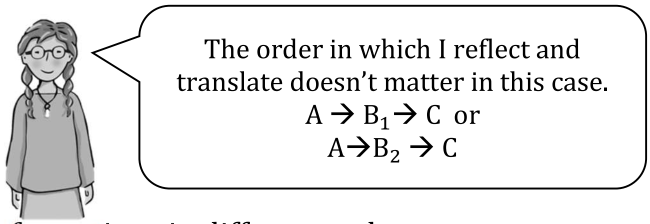
d) A reflection of the triangle in the line $y = 3$

3. A student is exploring the effect of combining a translation and a reflection.



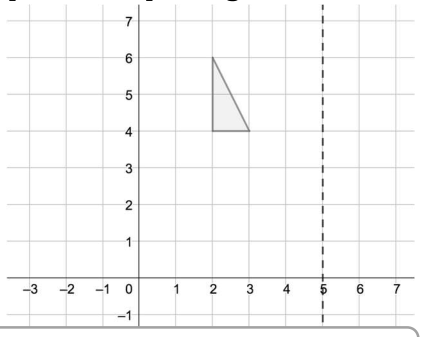
Reflect in the dotted line

Translate by the vector $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$



4. Explore completing the two transformations in different orders:

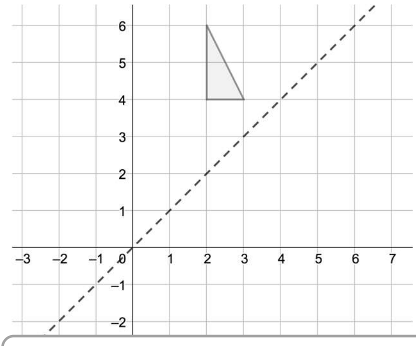
a)



Reflect in the dotted line

Translate by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

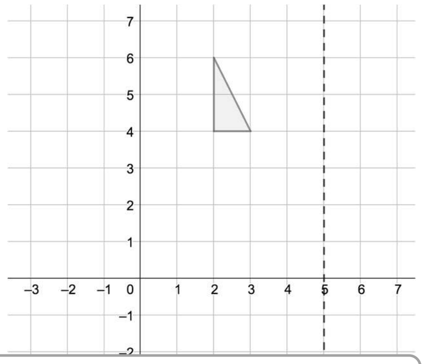
b)



Reflect in the dotted line

Translate by the vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

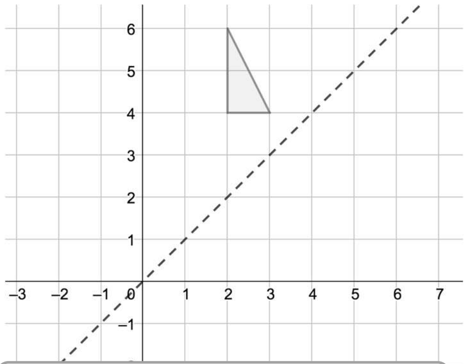
c)



Reflect in the dotted line

Translate by the vector $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

d)



Reflect in the dotted line

Translate by the vector $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

Questions for depth:

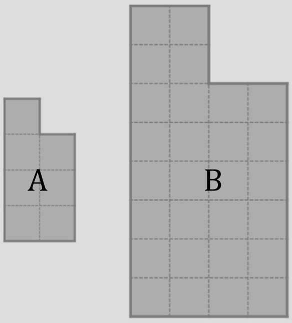
1. How far apart are the two options?

Option 1: A shape is translated by a vector $\begin{pmatrix} a \\ 0 \end{pmatrix}$ then reflected in the line $x = b$

Option 2: The same shape is reflected in the line $x = b$ then translated by a vector $\begin{pmatrix} a \\ 0 \end{pmatrix}$

Week 2 Session 3: Enlargement

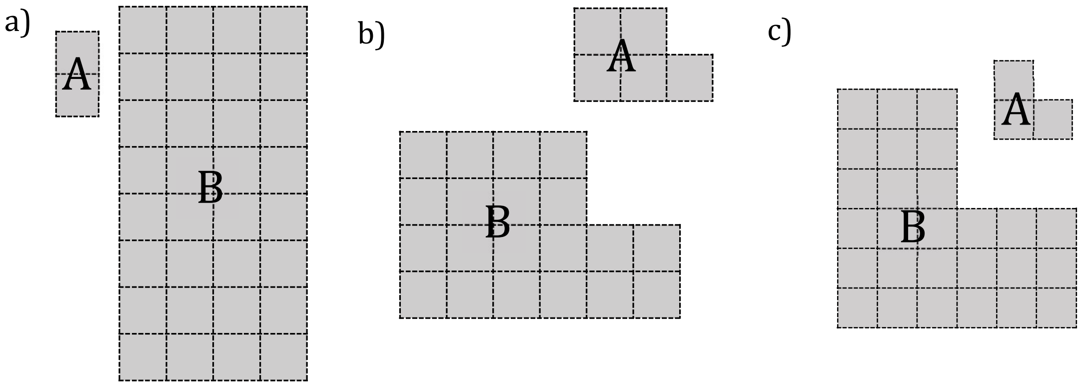
Concept Corner



$\frac{1}{2}$ | scale factor | enlargement | 2 | half

We can enlarge shapes using a ____________ ___________. The enlarged shape will have the same 'form' as the original shape. For example the transformation from A to B is an __________ of scale factor ____, the sides of the polygon are twice as long. The transformation from B to A is an enlargement of scale factor ____, the sides are ______ as long.

1.



i) State the scale factor of enlargement for each of the following transformations from A to B

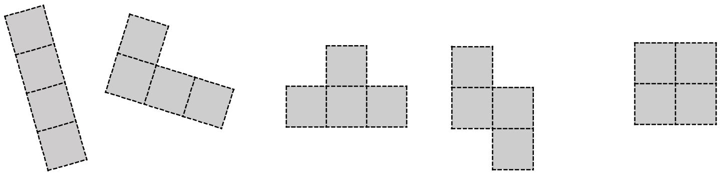
ii) State the scale factor of enlargement from B to A

2. On squared paper draw an enlargement of each shape:

a) with a scale factor of 2

b) with a scale factor of 3

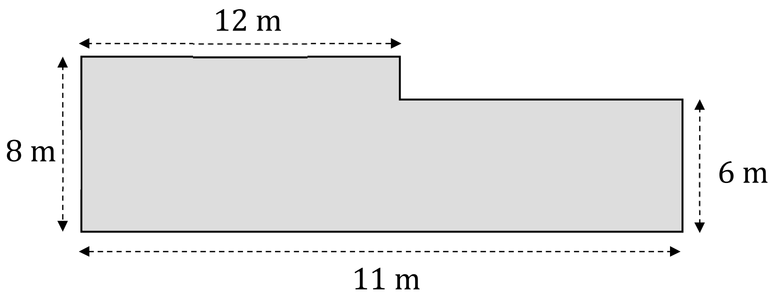
c) with a scale factor of 1



3. Find the perimeters of each of the shapes in Q1. What do you notice?

4.

Draw a **sketch** of the following shape after it has been enlarged by a scale factor of :

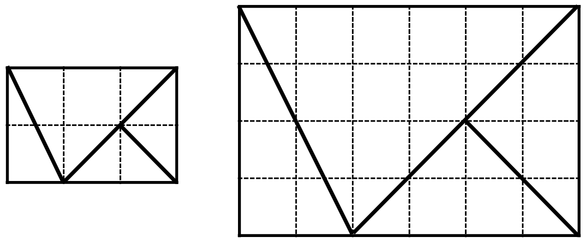


a) 2 b) $\frac{1}{2}$ c) 3

5. Find the perimeters of each of the enlarged shapes in Q4.

6.

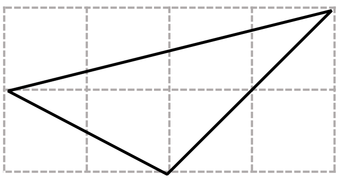
a) The 2 by 3 rectangle below was partitioned into 4 triangles and enlarged by a scale factor of 2:



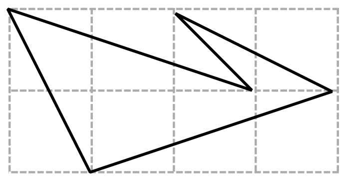
Draw a 2 by 3 rectangle on squared paper and partition it your own way, then enlarge it by a scale factor of 2.

7. Sketch each of the following shapes on square paper and then enlarge them by a scale factor of 3 :

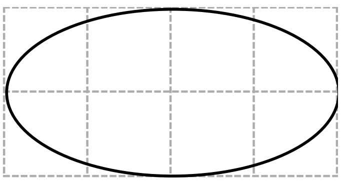
a)



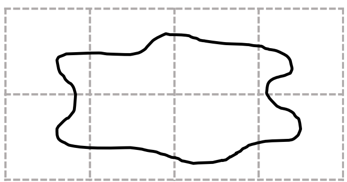
b)



c)



d)

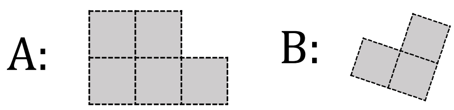


Questions for depth:

1. Hexagon A is enlarged by a scale factor of a and hexagon B by a scale factor of b. The perimeters of the two enlarged shapes are the same.

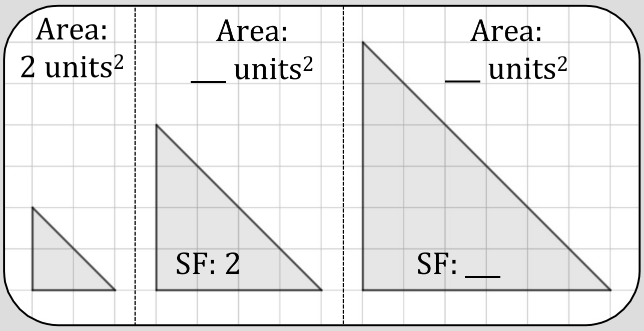
a) Suggest four possible values for a and b

b) Write an equation linking a and b



Week 2 Session 4: Enlargements and area

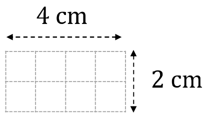
Concept Corner

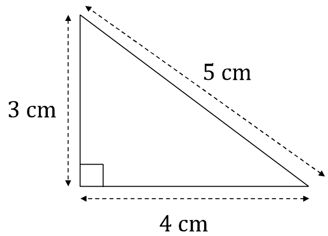


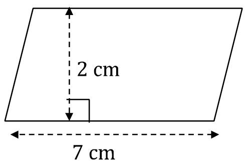
4 | 3 | changes | 8 | 18 | enlarge | scale factor

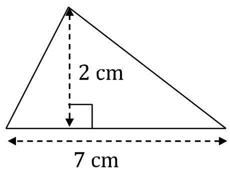
When we _______ a shape can affect the area. There is a relationship between the scale factor and how the area _______. The example above shows two different enlargements of the triangle on the left. If we enlarge by a _______ _______ of 2, the area becomes ____ times greater.

1. Find the area of the following shapes:

a) 

b) 

c) 

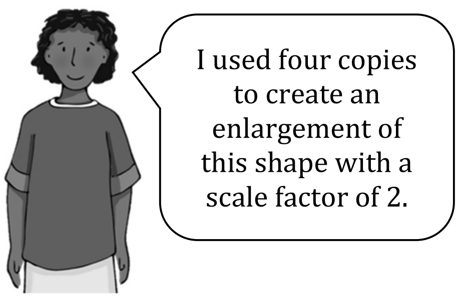
d) 

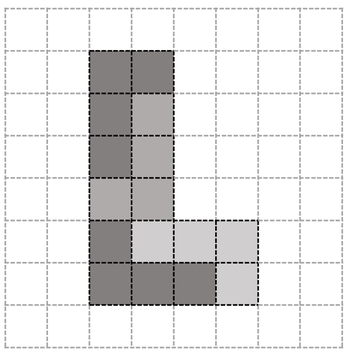
2.

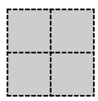
a) **Sketch** each of the shapes above following an enlargement of scale factor 3

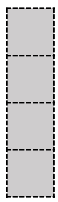
b) Find the area of the enlarged shapes, what do you notice?

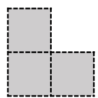
3. Hassan is trying to use copies of a shape to create an enlargement. Use four copies of each shape to enlarge them by a scale factor of 2.



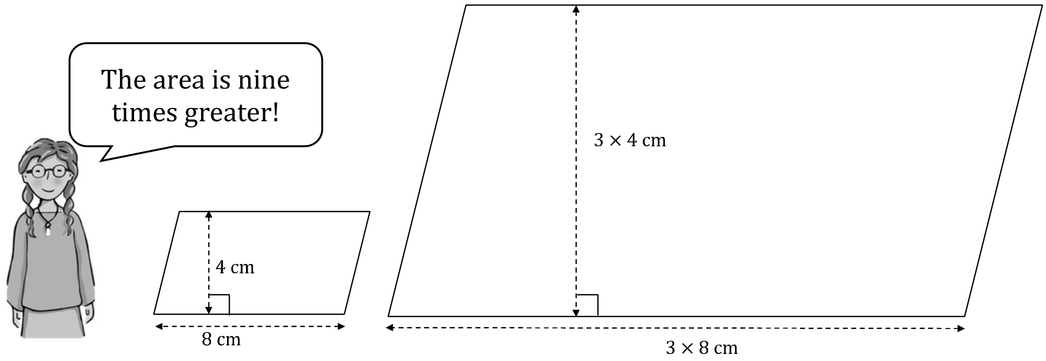


a) 

b) 

c) 

4. Rosie is comparing the area of a parallelogram before and after it has been enlarged:



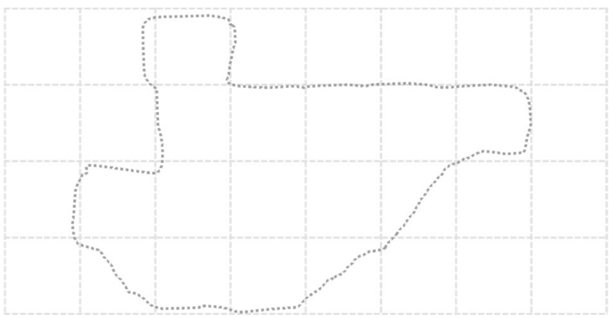
Area = $4 \times 8\ \text{cm}^2 = 32\ \text{cm}^2$

Area = $3 \times 4 \times 3 \times 8\ \text{cm}^2 = 9 \times 32\ \text{cm}^2$

Use a similar strategy to show many times greater the area is following an enlargement by scale factor:

a) 5 b) 12 c) n

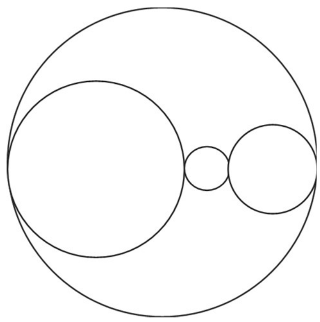
5. A map of an island is drawn on a square centimetre grid. The actual island is an enlargement of the map by a scale factor of approximately 2 000 000.



a) Estimate the length of the coast line.
b) Estimate the area of land of the island.

Questions for depth:

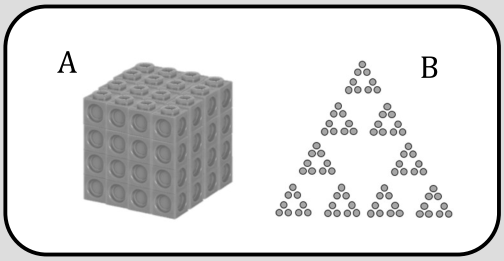
1. Three circles of radius $1\ cm$, $2\ cm\ and\ 6\ cm$ sit inside a larger circle. The four centres lie on the same horizontal line. Compare the area of the smallest circle to the three larger circles. How many times greater are they?



Week 3: Prime factorisation 1

Session 1: Indices

Concept Corner



4^3	index
3^4	cubed
4	squared

When numbers are written "*to the power of*" it is called ___________ notation.
So 5 × 5 can be written as 5^2, and is said '5 to the power of 2', or '5 _____________'.
6 × 6 × 6 × 6 can be written as 6^4, and is called 6 to the power of ___.

Diagram A shows 4 × 4 × 4 which can be written as ___ and is said '4 ___________'.
Diagram B shows 3 × 3 × 3 × 3 or ___.

1. Write the following calculations using index notation:

   a) 6×6

   b) $6 \times 6 \times 6 \times 6$

   c) $2 \times 2 \times 2 \times 2 \times 2$

2. Find the product of Q1 parts a) and b), write the answer in index notation.

3. Copy the equations below. Circle those that **are** true, and cross through those that aren't true.

$9^1 = 9$ $\quad$ $5^3 = 25 \times 5$

$5^3 = 3 \times 3 \times 3 \times 3 \times 3$ $\quad$ $10^2 = 2^{10}$

$8^3 = 24$ $\quad$ $8^4 = 8 \times 8 \times 8 \times 8$

4. Place the correct symbol (<, >, or =) between each pair of numbers

   a) 2^3 _____ 3^2

   b) 2^4 _____ 4^2

   c) 3^3 _____ 5^2

   d) 1^8 _____ 1^5

5. Organise the diagrams into the following groups:

2 to the power of 4:

A ____ ____

3 to the power of 3:

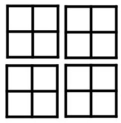
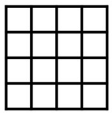
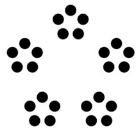
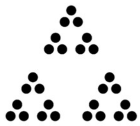
____ ____ ____

4 to the power of 2:

____ ____ ____

5 to the power of 2:

F ____ ____

2^4 A	4×4 B	 C	3^3 D
 E	5×5 F	4^2 G	 H
5^2 I	 J	 K	$2 \times 2 \times 2 \times 2$ L

6. Match the calculations on the left to the versions written in index form.

$2 \times 2 \times 7 \times 7 \times 7$	$5^3 \times 3$
$5 \times 3 \times 5 \times 3$	$2^2 \times 3^2 \times 5^2 \times 7$
$7 \times 7 \times 5 \times 5 \times 3 \times 3 \times 5 \times 7$	$3^2 \times 5^2$
$5 \times 5 \times 3 \times 3 \times 2 \times 2 \times 7$	$2^2 \times 7^3$
$5 \times 5 \times 5 \times 3$	$3^2 \times 5^3 \times 7^3$

7. $3^a > 100$. Find the smallest integer value of a.

8. 2^b is a square number. Find **three different** possible values for b.

Questions for depth:

1. Use the associative property of multiplication to write these calculations using index notation

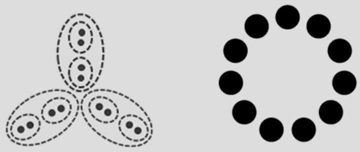
   a) 2 × 4

   b) 5 × 15

   c) 3 × 12

Week 3 Session 2: Prime factors

Concept Corner



5	can
12	cannot
11	prime

Prime numbers, like 5 and ____, can be multiplied together to make other numbers.

For example, 2s and 3s can be multiplied to make _____.

Prime numbers ___________ be made by multiplying smaller primes.

All non-prime numbers greater than 2 can be made by multiplying ____________ numbers, e.g. 70 = 2 × _____ × 7.

1. Copy and complete the frames to find ways of showing the numbers as products of **different** combinations of factors.

a) 24 = ___ × ___

24 = ___ × ___ × ___

24 = ___ × ___ × ___ × ___

b) 120 = ___ × ___

120 = ___ × ___ × ___

120 = ___ × ___ × ___ × ___

2. Copy and complete so that each equation shows a number as the product of **prime** factors.

a) 12 = 2 × 3 × ___

b) 20 = ___ × 2 × 5

c) 30 = 2 × ___ × 5

d) 36 = ___ × 2 × 3 × 3

e) 45 = 3 × ___ × ___

f) 54 = 2 × 3 × ___ × ___

3. Rewrite the following products so that only prime factors are used

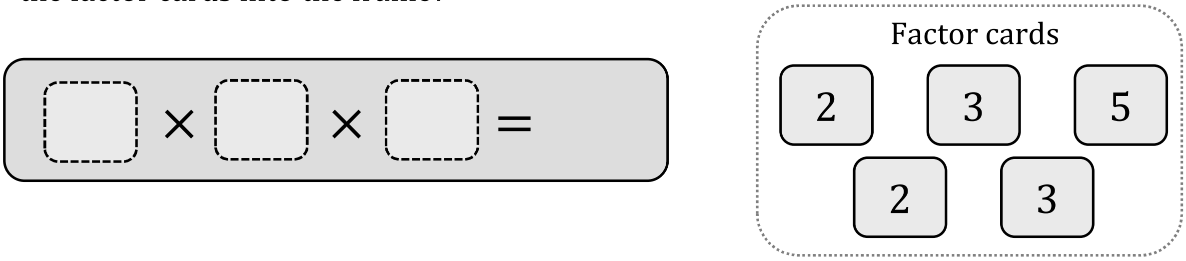
a) 24 = 2 × 3 × **4**

b) 48 = 2 × **24**

c) 60 = **4** × **15**

d) 72 = **4** × 2 × **9**

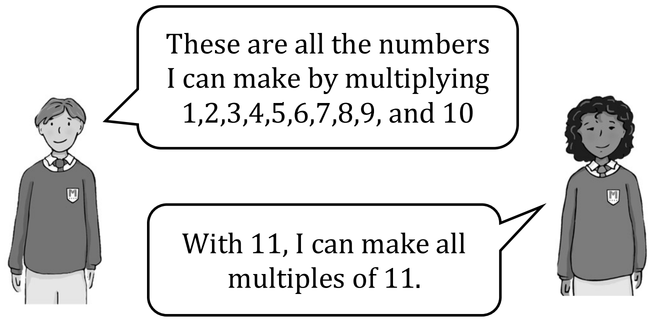
4. How many different products can you find by placing different combinations of the factor cards into the frame?



5. Gavin and Brenda have chosen a list of factors to multiply together.

   Gavin has shaded in the numbers he thinks he can make by multiplying these factors (he can use factors more than once in each multiplication).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



   a) Show how Gavin can make the following numbers by multiplying factors from his list:

      $i)\ \ 28\ =$ $\qquad$ $ii)\ \ 63\ =$ $\qquad$ $iii)\ \ 42\ =$

   b) Do you agree that Brenda will be able to make all multiples of 11? Explain why or why not.

   c) Gavin thinks he could remove factors from the list and **still make** all the shaded numbers.

      Which factors could he remove? Why?

Questions for depth:

The grid has been shaded in the same way as in question 4) above, using a list of **seven** factors.

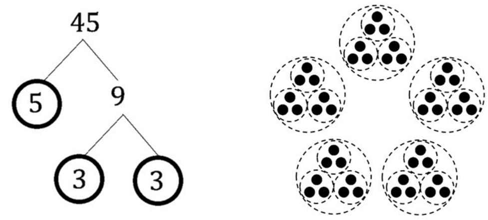
1. What was the list of factors?

2. What are the next three numbers greater than 100 that can be shaded?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Week 3 Session 3: Prime factorisation

Concept Corner



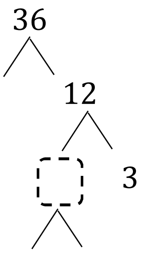
3 | compound | product | index

Numbers such as 18 and 45 are called ___________ integers and can be built as the ___________ of prime factors.

e.g. 45 can be written as _____ × 3 × 5, or 3^2 × 5 in ___________ notation.

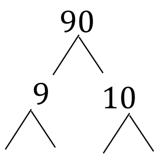
1. Copy and complete the prime factorisation trees and write the numbers as products of prime factors.

a)



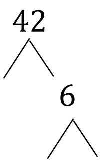
36 = __________

b)



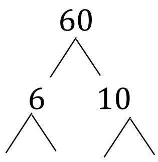
90 = __________

c)



42 = __________

d)



60 = __________

2. Write the following numbers as products of their prime factors.:

a) 72

b) 175

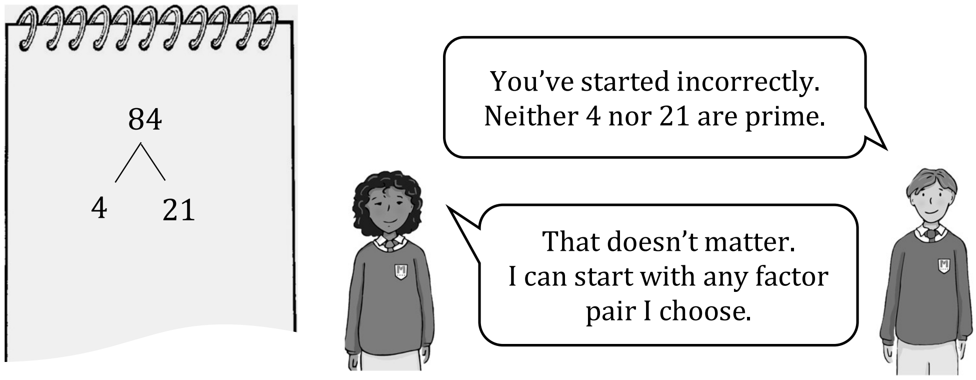
c) 144

d) 1750

e) 350

f) 216

3. Brenda is writing out the prime factor tree for 84. Gavin is commenting on her working.



Who do you agree with? Explain your answer. You may use examples to help.

4. Look at your answers to question 2. and answer the questions below.

a) Compare the prime factors of parts a), c) and f). What do you notice?

b) Compare the prime factors of parts b), d) and e). What do you notice?

c) Write down the prime factors of 720. Explain how you can use your answer to 2a) to help.

5. The cards below show three different numbers ***a***, ***b***, ***c*** and ***d*** written as products of their prime factors.
Decide if the statements below are **true or false.**

$a = 2 \times 3^2 \times 5^2$	$b = 2 \times 3^2 \times 5^3$
$c = 2^3 \times 3^2 \times 5^3$	$d = 2^2 \times 3^2 \times 5^2$

a) ***d*** is twice the value of ***a***

b) ***c*** is three times the value of ***b***

c) ***c*** is ten times the value of ***d***

d) ***b*** is less than ***d***

Questions for depth:

1. Two numbers, m and n have been written as the products of their primes, where x, y and z are **different** prime numbers. Decide whether the statements below are true or false.

a) ***m*** and ***n*** can be equal in value
b) ***m*** and ***n*** are square numbers
c) ***m*** and ***n*** are multiples of $(x^2 \times y^2 \times z^2)$

$m = x^2 \times y^3 \times z^4$

$n = x^4 \times y^3 \times z^2$

Week 3 Session 4: Using the prime factorisation

Concept Corner

Prime factorisations can be used to deduce factors of a number:

$$84 = 2^2 \times 3 \times 7 = 12 \times 7$$

12	prime	
7	pair	
21	4	21

In the example above the _________ factors 2, 2, and 3 have been multiplied together showing us that ____ and ____ are a factor ________ of 84.

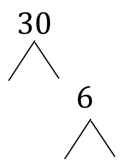
Using a similar approach another factor pair of 84 can be made:

$2 \times 2 =$ ____ $\qquad 3 \times 7 = 21$

So, 4 and ____ are a factor pair of 84.

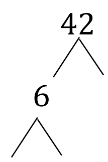
1. Copy and complete the prime factorisation trees and write the numbers as products of prime factors.

a)



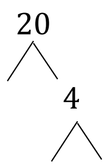
30 = __________

b)



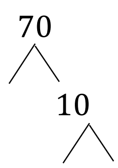
42 = __________

c)



20 = __________

d)



70 = __________

2.

a) Use your answers to question 1 to help work out how many factors each of the numbers 30, 42, 20 and 70 has.

b) What's the same or different about the **number of factors** each number has? Explain why this is the case.

c) Find a number that has the same number of factors as 20.

3. The numbers in the box below have been written as the product of their prime factors. Use this information to help you answer the questions below.

72 $= 2^3 \times 3^2$	**75** $= 3 \times 5^2$
168 $= 2^3 \times 3 \times 7$	**30** $= 2 \times 3 \times 5$
245 $= 5 \times 7^2$	**42** $= 2 \times 3 \times 7$

a) 75 = 15 × _____

b) Does 168 have a factor that is a square number?

c) Two of the numbers multiply to make a square number, which two?

d) The product of all of the numbers in the box is 280052640000. Write 280052640000 as a product of prime factors

4. $2 \times 3^3 \times 5 = 270$

Every factor pair for 270 will have one even factor and one odd factor.

Do you agree or disagree with Nicola? Explain why.

5. $210 = 2 \times 3 \times 5 \times 7$ $\quad$ $220 = 2^2 \times 5 \times 11$ $\quad$ $230 = 2 \times 5 \times 23$

Which of these numbers has the most factors? Which of these numbers has the least factors?

6. Which of the numbers below has the **greatest** factor?

$315 = 3^2 \times 5 \times 7$ $\quad$ $210 = 2 \times 3 \times 5 \times 7$ $\quad$ $220 = 2^2 \times 5 \times 11$

Questions for depth:

1. Look at the number below written as the product of its prime factors.

$$42336 = 2^5 \times 3^3 \times 7^2$$

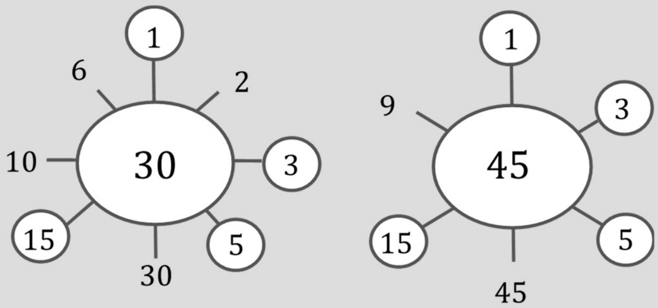
a) $42336 \div a$ results in an odd integer. What are the greatest and least possible values of a?

b) $42336 \times b$ results in a cube number. What are the greatest and least possible values of b?

Week 4: Prime factorisation 2

Session 1: Highest common factor

Concept Corner



15 | HCF | factors | common

The highest common factor, abbreviated as the _____, can be found by listing the ____________ of each number.

We can use the diagram to see that the HCF of 30 and 45 is ____.

1. List all of the factors of:

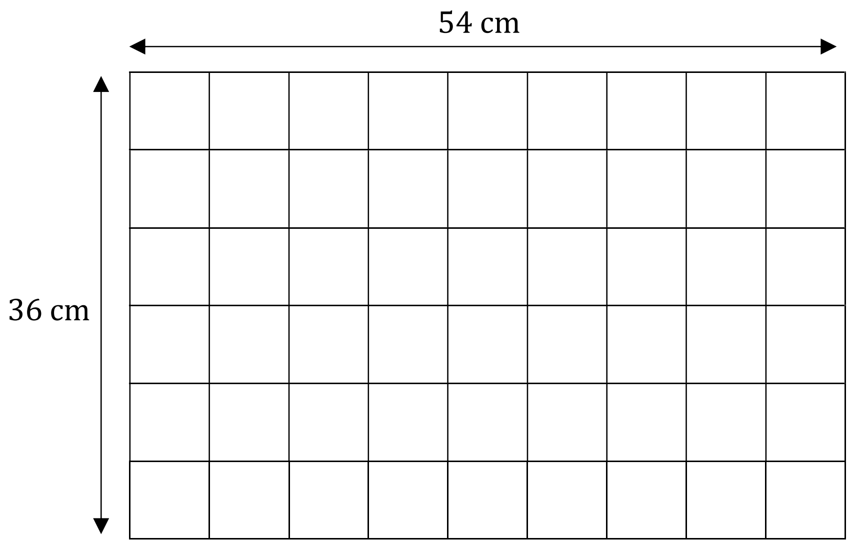
a) 28 b) 54 c) 36 d) 19

2. Identify **all the common factors** of:

a) 28 and 54 b) 28 and 36 c) 28 and 19 d) 54 and 36

3. Find the HCF for the pairs in Q2.

a) What size squares has this student split the rectangle in to?
b) What other squares size squares can you split it into?



4.

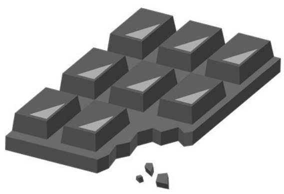
a) Find three pairs of numbers that have a HCF of 12.

b) What other factors do the pairs have in common?

5. Find the HCF of the following:

a) 12 and 18	b) 12 and 30	c) 12 and 42	d) 12 and 54
e) 16 and 24	f) 40 and 24	g) 64 and 24	h) 88 and 24

6. A shop sells boxes of chocolate. In total there are 252 dark chocolates and 180 milk chocolates. If every box is identical, how many boxes could there be?



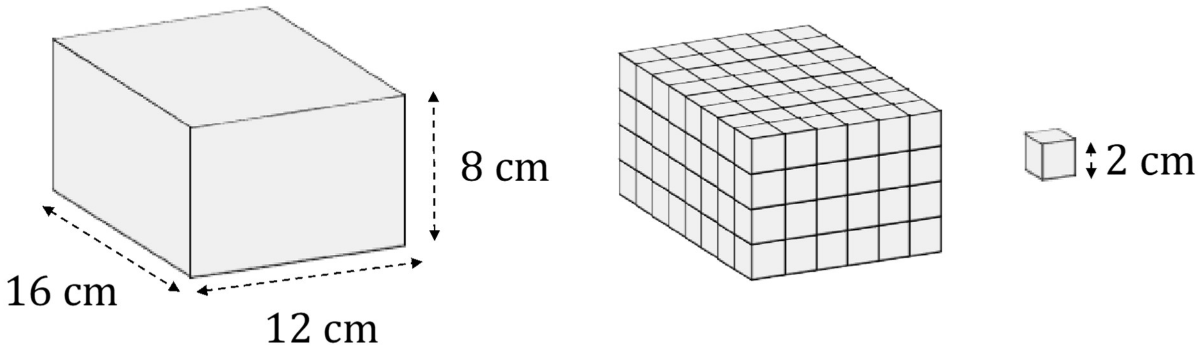
7. A pair of distinct two-digit numbers have a common factor of 16.

a) Find three possible pairs

b) Find all the possible values for the HCF.

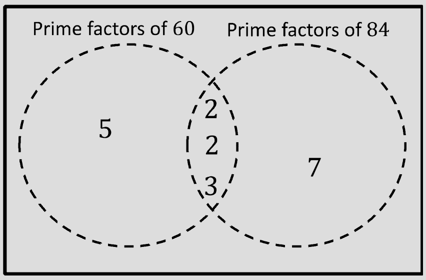
Questions for depth:

1. How else can you split the cuboid into identical cubes? Explore this for different sized cuboids.



Week 4 Session 2: More highest common factor

Concept Corner



$60 = 2 \times 2 \times 3 \times 5$
$84 = 2 \times 2 \times 3 \times 7$

60 | prime | 84 | product | 6 | 12 | HCF

Writing numbers as a _________ of their ______ factors helps to reveal common factors. The Venn diagram shows the prime factors of _____ and _____. We can see that ____ is a common factor and _____ is the ____.

1. Write each of the numbers as a product of primes:

a) 130 b) 104 c) 56 d) 308

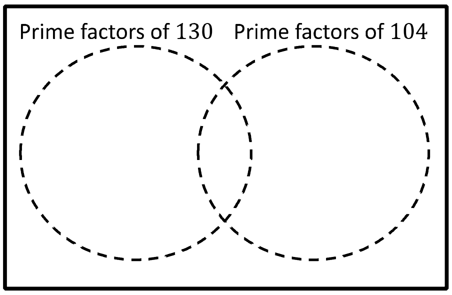
2. Given that $1680 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7$

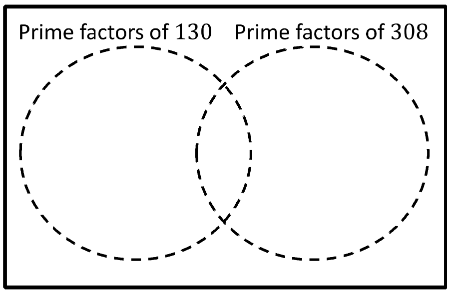
a) Decide whether each of the following are factors of 1680:

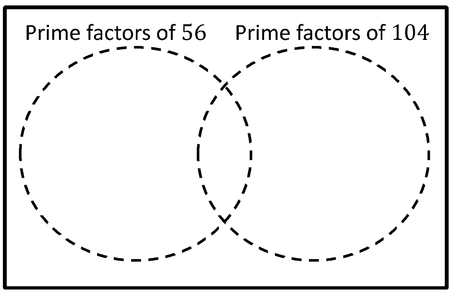
i. 2 ii. 7 iii. 14 iv. 13
v. 8 vi. 11 vii. 32 viii. 48

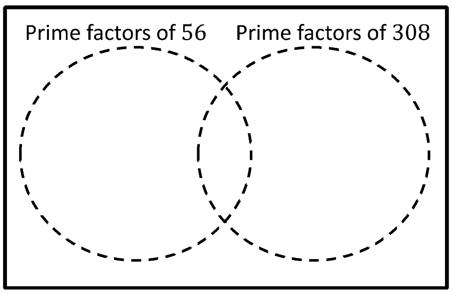
b) Which of the numbers from Q1 is a factor of 1680?

3. Copy and complete the Venn diagrams:

a)


b)


c)


d)


4. Find the highest common factor of each pair

a) 130 and 104

b) 130 and 308

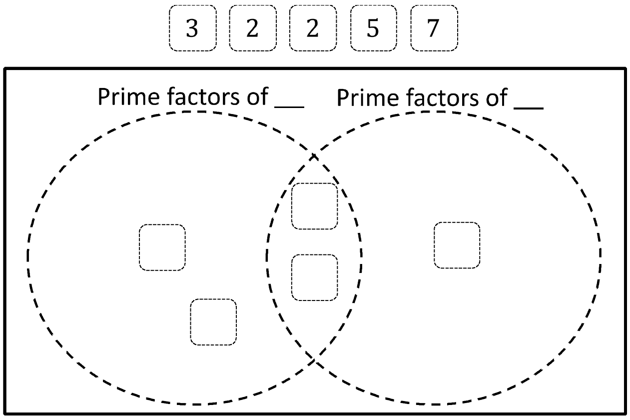
c) 56 and 104

d) 56 and 308

e) 130 and 56

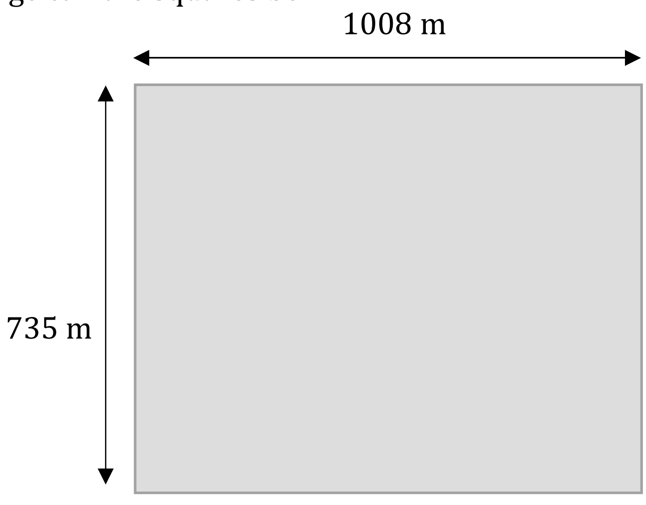
f) 308 and 104

5. Place the primes into the Venn diagram. Find the possible pairs of numbers and their highest common factors:



6. Find three examples of a pair of numbers greater than 1000 that have a HCF of 72.

7. A rectangular field needs to be divided in to equally sized, square plots of land. How large can the squares be?



Questions for depth:

1. Compare the **HCF of a and b** with the **HCF of a and $a + b$** . Select your own values for a and b. What do you notice? Will this always be true? Explain your answer.

Week 4 Lesson 3: Lowest common multiple

Concept Corner

Multiples of 3: 3 , 6 , 9 , (12), 15 , 18 , 21 , (24), 27 , 30

Multiples of 4: 4 , 8 , (12), 16 , 20 , (24), 28 , 32 ...

3	lowest
4	multiples
12	LCM

The __________ common multiple, abbreviated as the _____, can be found by listing the ___________ of each number.

We can see that the LCM of 3 and 4 is ____.

1. List the first 12 multiples of:

a) 12 b) 9 c) 7 d) 21

2. Identify the lowest common multiple of:

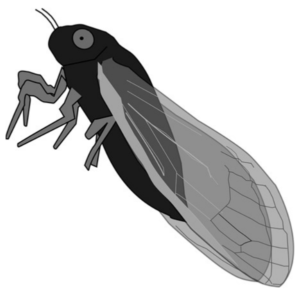
a) 12 and 9 b) 12 and 7 c) 12 and 21

d) 21 and 7 e) 21 and 9 f) 9 and 7

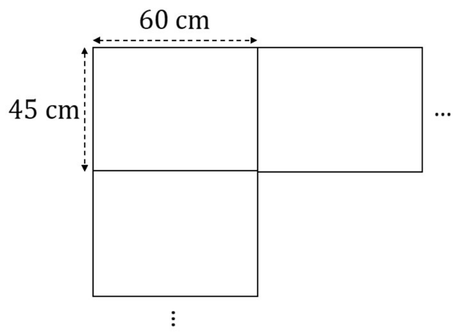
3. Find three examples of a pair of numbers that have a LCM of

a) 15 b) 21 c) 30 d) 36

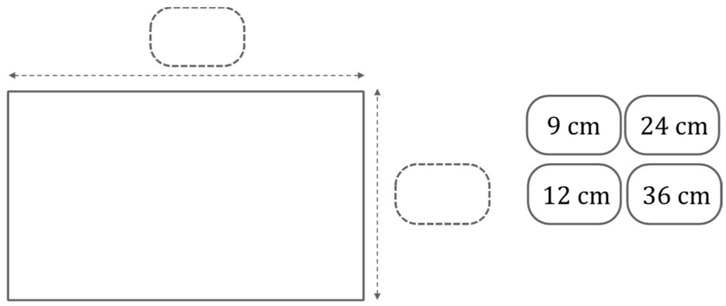
4. A cicada lives underground and appears every 17 years. A predator of the cicada appears every 4 years. If they hide at the same time, how long will it be before they appear again at the same time?



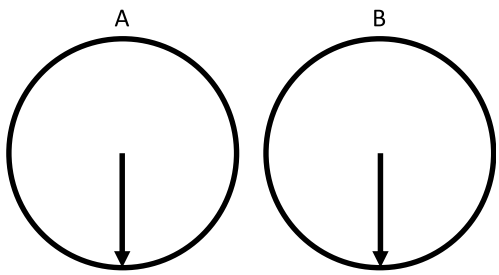
5. Charlie is trying to create a squares by tessellating the rectangle below. What size squares can he make?



6. Select two side lengths from the numbers below. Explore the different squares you can make by tessellating the rectangle:



7. **A** takes 12 seconds to do a full turn, B takes 21 seconds. They repeatedly spin in the same direction starting in the position shown:



a) How much time passes before A and B return to their starting position at the same time?

b) Will your answer change if they spin in opposite directions? If so, how?

Questions for depth:

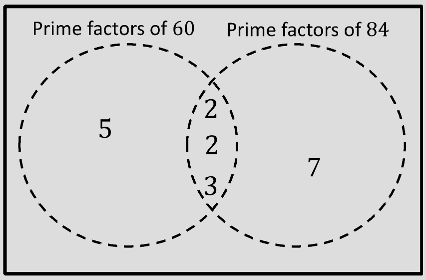
1. Following on from the situation in Q7:

a) When is the first time that the arrows point in the same direction?

b) Can the arrows both point upwards at the same time?

Week 4 Session 4: More lowest common multiple

Concept Corner



$60 = 2 \times 2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7$

84	prime
60	product
420	LCM

Writing numbers as a _________ of their ______ factors helps you to find the lowest common multiple.

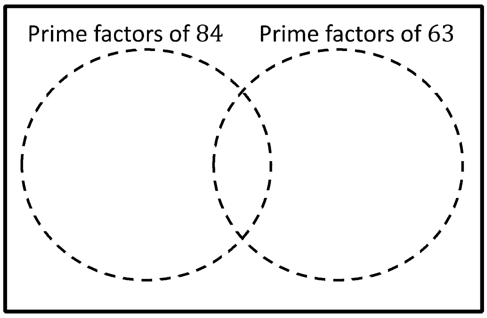
The Venn diagram shows the prime factors of 60 and 84. We can see that the _____ is ____ × 7 = ____ × 5 = ____

1. Write each of the numbers as a product of primes:

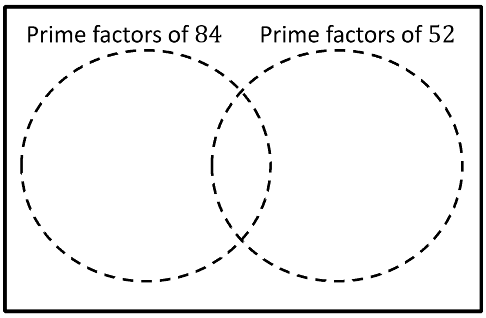
a) 63 b) 84 c) 52 d) 36

2. Copy and complete the Venn diagrams:

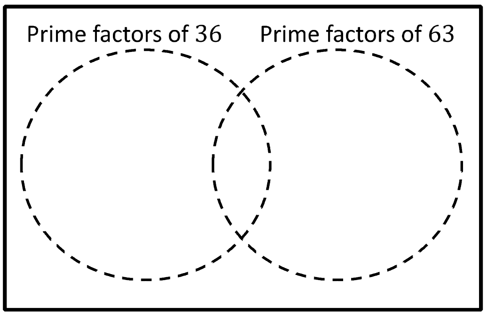
a)



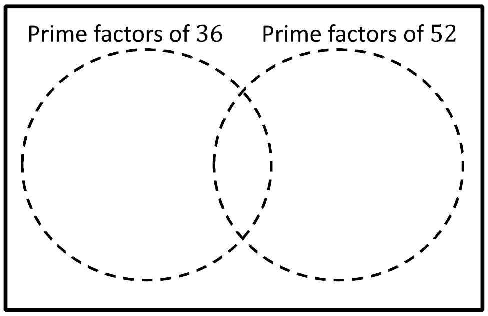
b)



c)



d)

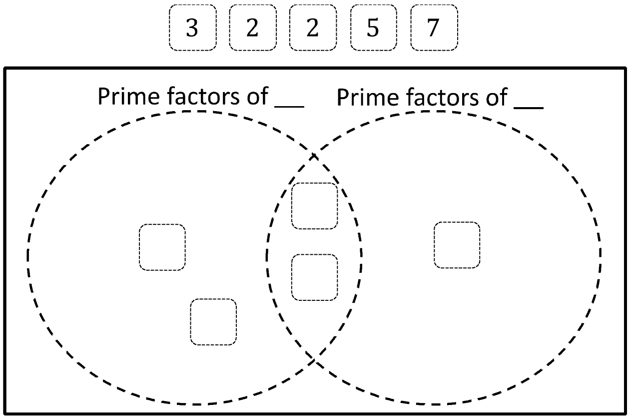


3. Find the lowest common multiple of each pair:

a) 84 and 63 b) 84 and 52 c) 36 and 63

d) 36 and 52 e) 84 and 36 f) 63 and 52

4. Place the primes into the Venn diagram. Find the possible pairs of numbers and their lowest common multiples:

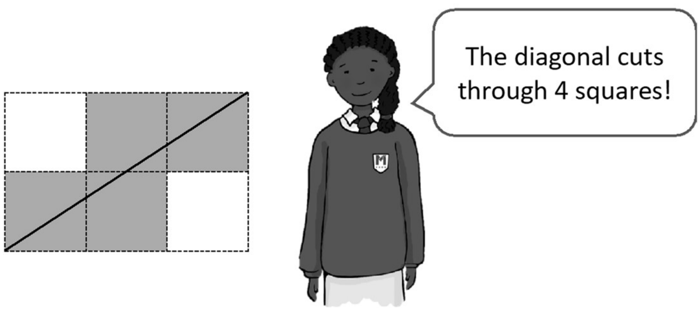


5. Find examples of pairs of two digit numbers with a LCM greater than 1000.

Questions for depth:

1. Find the product of the highest common factor and the lowest common multiple for different pairs of numbers. What do you notice? Why does this happen?

2. Neda is investigating the number of squares that a rectangle's diagonal crosses. Investigate how many squares are crossed for different rectangles. What do you notice?



Week 5: Fractions

Session 1: Part of a whole

Concept Corner

$\frac{1}{3}$	whole	numerator
$\frac{2}{3}$	denominator	
one third	two thirds	

We can use fraction notation to describe part of a ________.

The ____________ represents the total number of equal parts, the ____________ represents the number of parts we are describing.

E.g. if  is the whole.

The shaded section of  represents ____ or "______ _______".

The unshaded section of  represents ____ or "______ _______".

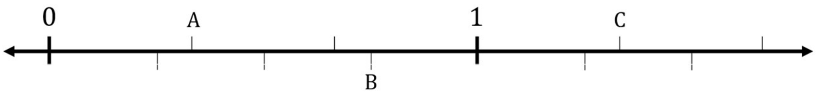
1. Find the matching pairs

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{4}{3}$ d) $\frac{3}{2}$ e) $\frac{3}{4}$

Four thirds	Two thirds	Three halves	Two fifths	Three quarters

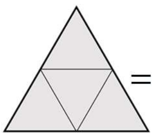
2. a) Write the values of the marked points of the number line:

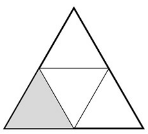
i) A =
ii) B =
iii) C =

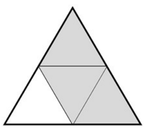


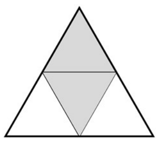
b) Suggest a value in fraction notation that lies between:

i) 0 and A: _________ ii) A and B: _________ iii) 1 and C: _________

3. If  = 1 , write a fraction to represent the value of the shaded section:

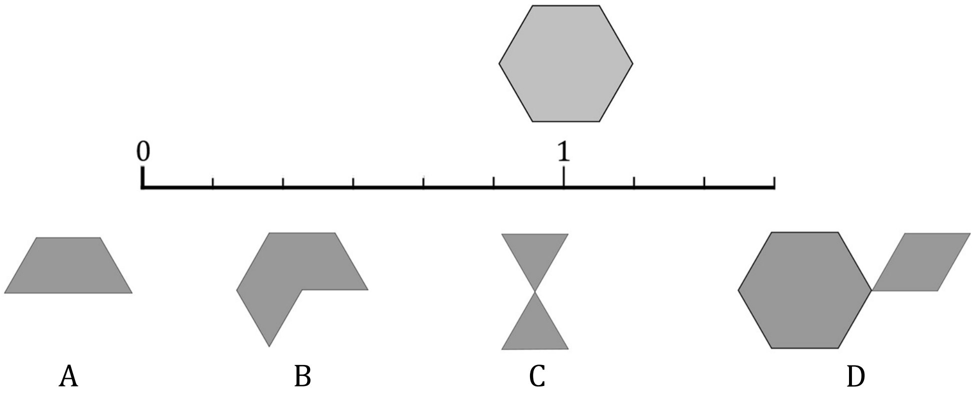
a) 

b) 

c) 

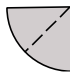
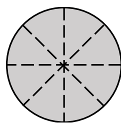
Mathematics Mastery Ark

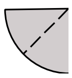
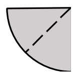
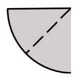
4. Draw a copy of the numberline and identify where the shapes should be placed:

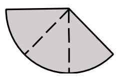
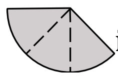
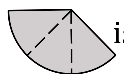


5. Sketch a diagram to complete the statement:

e.g.

 is $\frac{1}{4}$ of 

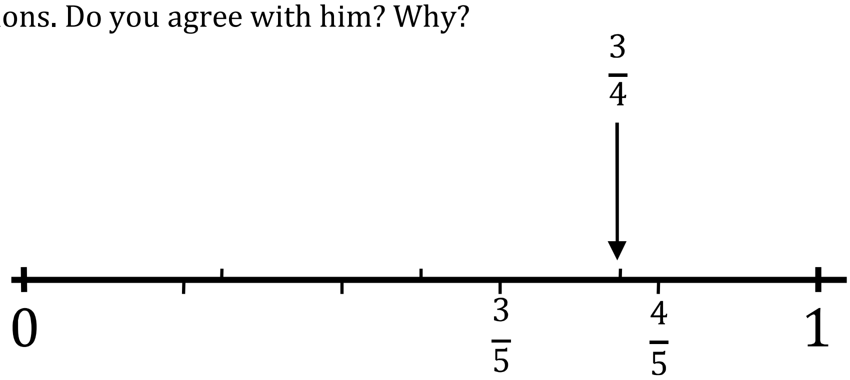
a)  is $\frac{1}{2}$ of ____________ b)  is $\frac{1}{3}$ of ____________ c)  is $\frac{2}{3}$ of ____________

d)  is $\frac{1}{2}$ of __________ e)  is $\frac{3}{4}$ of __________ f)  is $\frac{1}{3}$ of __________

Questions for depth:

1. Tom thinks that that you will always be able to find a fraction 'in between' two other fractions. Do you agree with him? Why?

e.g.



Week 5 Session 2: Fractions of measure

Concept Corner

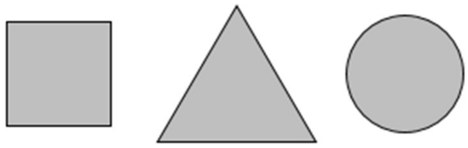
area | fractions | $\frac{1}{3}$ L | time

We can use ________ when describing units of measure.

Units of measure can be used to describe: _____, ______, angles, volume etc.

For example if a cube holds 1 L of water 

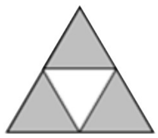
Here  we have _____ of water.

1. Each of the following shapes has an area of $1m^2$ 

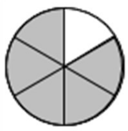
   a) Find the area of:

   i) 

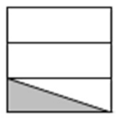
   ____

   ii) 

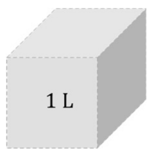
   ____

   iii) 

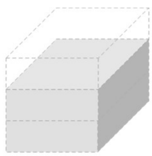
   ____

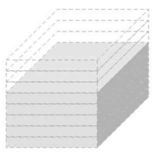
   iv) 

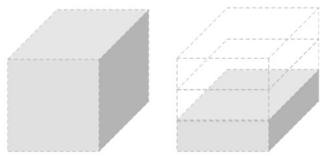
   ____

2. A cube can hold 1L of water: 

. How much water is shown below?

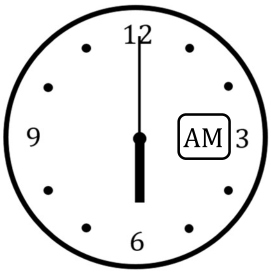
a)  ___ L

b)  ___ L

c)  ___ L

3. Billy and Tommy each have 1 L water bottles, they are half full. They pour the water into a 3 L container, have fraction of the 3 L container is full?

4. This clock shows 06:00.



   a) How many minutes will have past when the minutes hand has travelled $\frac{1}{4}$ of the way around the clock?

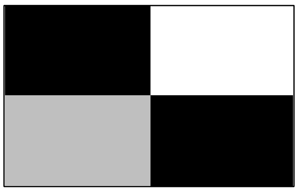
   b) What fraction of the clock will the hour hand have travelled by 10:00?

   c) What fraction of the clock will the **hour hand** have travelled when the minute hand has travelled all the way around the clock?

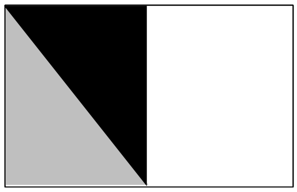
   d) What fraction of the clock will the **hour hand** have travelled when the minute hand has travelled half way around the clock?

5. If each flag has an area of $1m^2$, find the areas of each colour in fraction notation:

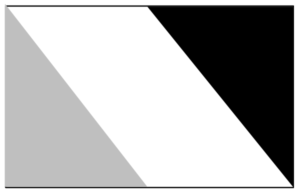
a)



b)



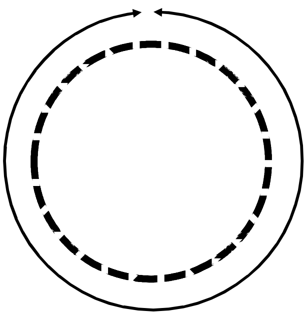
c)



6. Sketch three flags where $\frac{1}{4}$ of the area is white , $\frac{1}{4}$ is black and $\frac{1}{2}$ is grey.

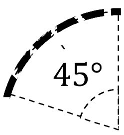
Questions for depth:

1. A 1 m length of rope is made into a circle. Find the length of each section of rope:

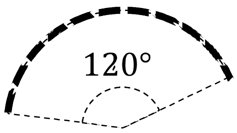


1 m

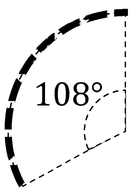
a)



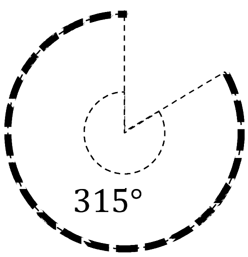
b)



c)



d)



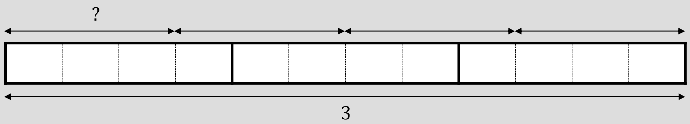
What angle is needed for a rope of length $\frac{3}{8}$ m?

Week 5 Session 3: Fair shares

Concept Corner

We can use _________ notation to describe a __________.

For example ___ ÷ ___ = ____



4

3

$\frac{3}{4}$

fraction

division

1. State the value of the shaded and unshaded sections:

a) 

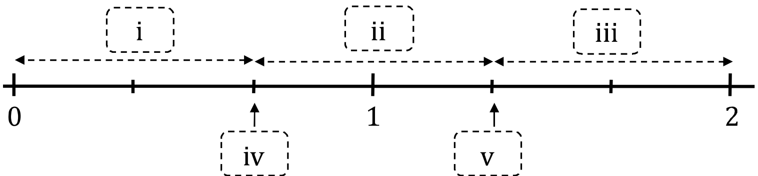
b) 

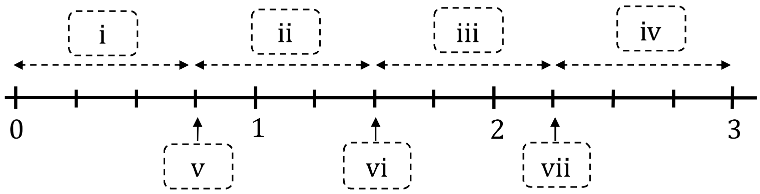
c) 

d) 

e) 

2. Find the missing numbers below:

a) 

b) 

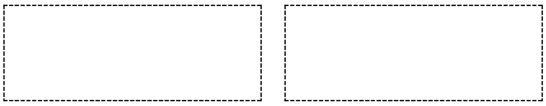
3. Write the following questions in fraction notation

a) 2 ÷ 3

b) 3 ÷ 4

c) 5 divided by 3

4. A group of friends are sharing 2 chocolate bars. What fraction of a chocolate bar do they each get if …



a) … there are 3 friends?

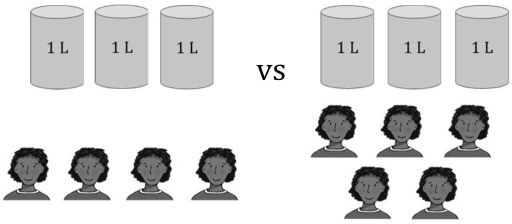
b) … there are 5 friends?

c) … there are 7 friends?

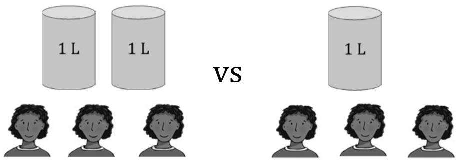
Sketch diagrams to represent each situation

5. In each situation decide which group gets more soda per person.
How much do they get each?

a)

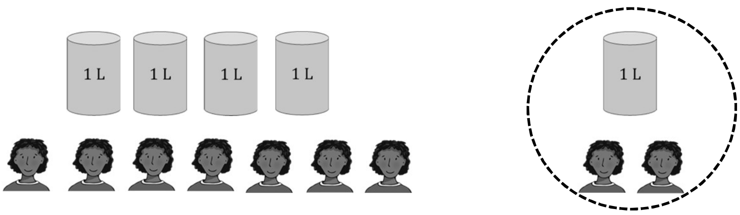


b)



c)

6. A group of 7 people plan to share 4L of soda. Two people join the party and bring 1 L of soda. Does the amount of soda per person increase or decrease?



Questions for depth:

1. n people plan to share 10 chocolate bars.

a) How much does each person get?

2 more people join the group and bring a chocolate bar with them…

b) When does the amount of chocolate per person increase? When does it decrease?

Week 5 Session 4: Equivalence

Concept Corner

Two fractions are said to be ____________ if they represent the same value.

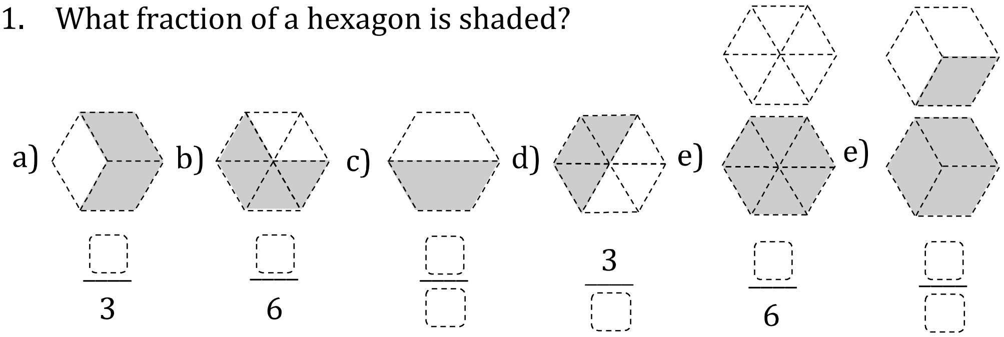
For example we can see that _____ = _____ so they are equivalent. Similarly, _____ = _____ so they are equivalent too.

$\frac{2}{3}$ $\frac{2}{6}$ $\frac{1}{3}$ $\frac{4}{6}$

equivalent

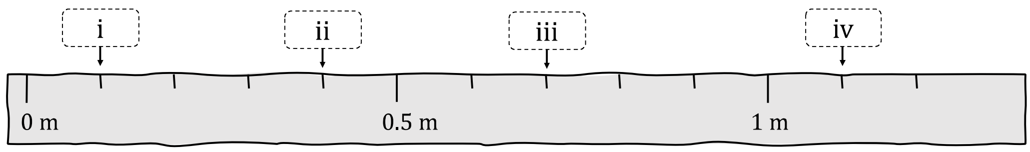
$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

1. What fraction of a hexagon is shaded?



2.

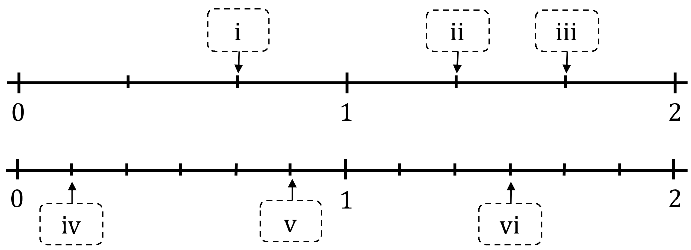
a) Find each of the marked decimal values on this tape measure:



b) Now write each of the answers in the form: $\frac{\square}{10}$

3.

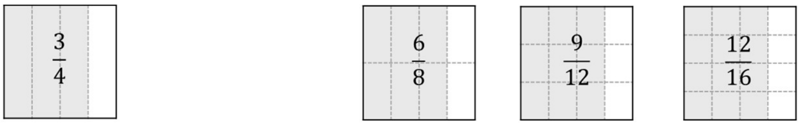
a) Find the marked numbers:



b) Use the number lines to find different ways to complete the following:

$$\frac{\square}{3} = \frac{\square}{6}$$

4. Here are three fractions that are equivalent to $\frac{3}{4}$:



Draw your own diagrams to show three fractions equivalent to: a) $\frac{2}{3}$ b) $\frac{4}{5}$ c) $\frac{4}{3}$

5. Copy and complete each of the following:

a) $\frac{1}{2} = \frac{\square}{10} = \square.\square$ b) $\frac{1}{5} = \frac{\square}{10} = \square.\square$ c) $\frac{2}{5} = \frac{\square}{10} = \square.\square$

d) $\frac{\square}{5} = \frac{12}{10} = \square.\square$ e) $\frac{\square}{\square} = 0.7$ f) $\frac{12}{30} = \frac{\square}{10} = \square.\square$

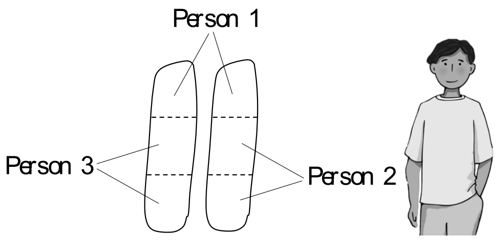
g) $\frac{36}{20} = \frac{\square}{10} = \square.\square$ h) $\frac{8}{25} = \frac{\square}{100} = \square.\square\square$ i) $\frac{\square}{50} = \frac{\square}{100} = 0.06$

6. Charlie has shown how he would share two sausages between three people.

a) What fraction of a sausage does each person get?

b) Draw a similar diagram to show the following.

What fraction of a sausage does each person get?



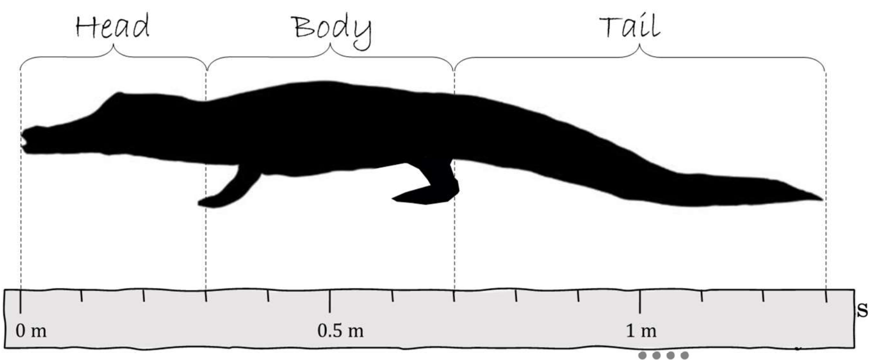
i) 4 sausages between 6 people ii) 4 sausages between 3 people iii) 8 sausages between 6 people

Questions for depth:

1.
a) What fraction of the crocodile's total length is its:

i) head? ii) body? iii) tail?

b) The world's longest recorded crocodile, Lolong, was 6.17 m long! Assuming it had the same proportions, find the approximate length of its head, body and tail.



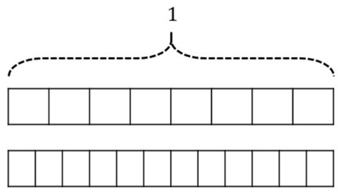
Week 6: Fractions 2

Session 1: Comparing fractions

Concept Corner

We can use our understanding of the ___________ and denominator to __________ fractions. This diagram shows that $\frac{1}{8}$ ____ $\frac{1}{12}$.

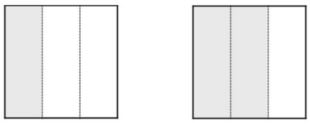
We can also use this to compare fractions as a distance from another point. For example, $\frac{11}{12} > \frac{7}{8}$ since $\frac{11}{12}$ is closer to 1. We can also see that $\frac{7}{12}$ _____ $\frac{5}{8}$ when we compare their distances from $\frac{1}{2}$.



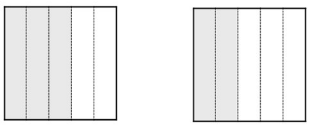
>	numerator
<	compare

1. Decide which fraction is greater, copy and complete the inequality with < or > .

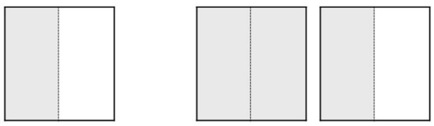
a) $\frac{1}{3} \square \frac{2}{3}$



b) $\frac{3}{5} \square \frac{2}{5}$

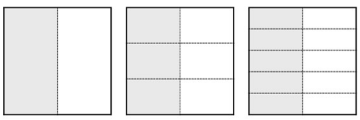


c) $\frac{1}{2} \square \frac{3}{2}$



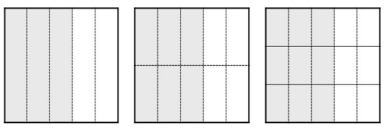
2. Use the diagram to find three equivalent fractions:

a)



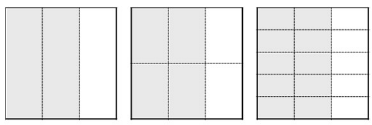
$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

b)



$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

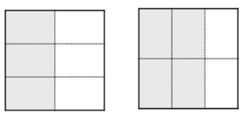
c)



$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

a) Copy and complete the following:

$\frac{1}{2} = \frac{\square}{6}$ and $\frac{2}{3} = \frac{\square}{6}$ so $\frac{1}{2} \square \frac{2}{3}$



b) Compare $\frac{2}{3}$ and $\frac{3}{5}$ using a similar strategy

3. Copy and complete each inequality with < or > :

a) $\frac{1}{5} \square \frac{2}{15}$

b) $\frac{3}{7} \square \frac{3}{8}$

c) $\frac{3}{4} \square \frac{11}{16}$

d) $-\frac{1}{2} \square -\frac{1}{4}$

4. Which of these fractions is:

a) Closest to 1

b) Closest to $\frac{1}{2}$

c) Largest

$\frac{3}{4}$ $\quad \frac{8}{7}$ $\quad \frac{11}{10}$ $\quad \frac{1}{13}$ $\quad \frac{7}{8}$ $\quad \frac{2}{3}$

5.

a) Fill in the blanks

i) $\frac{3}{\square} = \frac{1}{2}$

ii) $\frac{2}{3} = \frac{6}{\square}$

iii) $\frac{5}{\square} < \frac{1}{2}$

iv) $\frac{2}{3} > \frac{\square}{18}$

b) Find a value that satisfies both of the inequalities for part a iii) and iv)

Questions for depth:

1. How many ways can you complete the following inequality?

$$\frac{3}{4} < \frac{\square}{16} < \frac{7}{8}$$

Design your own inequality problem using the following template:

$$\frac{\square}{\square} < \frac{\square}{\square} < \frac{\square}{\square}$$

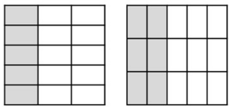
Week 6 Session 2: Common denominators

Concept Corner

We can systematically compare two fractions by finding a ____________ denominator.

This also helps us to accurately describe the _____________ between fractions.

e.g. $\frac{2}{5}$ is $\frac{1}{15}$ ________ than $\frac{1}{3}$



$\frac{1}{3} = \frac{5}{15}$ $\frac{2}{5} = \frac{6}{15}$

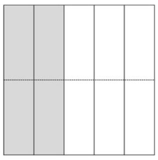
$$\frac{2}{5} > \frac{1}{3}$$

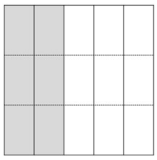
greater

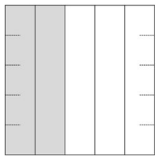
common

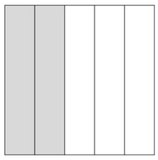
difference

1. Use these diagrams to calculate equivalent fractions for $\frac{2}{5}$

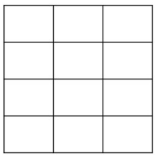
a)  $\frac{4}{\square}$

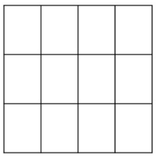
b)  $\frac{\square}{15}$

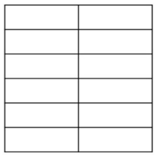
c)  $\frac{\square}{25}$

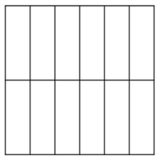


2. a) Shade in the diagrams to show

i) $\frac{2}{3}$ 

ii) $\frac{3}{4}$ 

iii) $\frac{1}{2}$ 

iv) $\frac{5}{6}$ 

b) Write each of the fractions in part a) as $\frac{\square}{12}$

c) Write each of the fractions in part a) in order from smallest to greatest.

3. a) Fill in the blanks:

i) $\frac{1}{4} = \frac{6}{\square}$

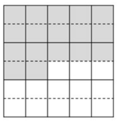
ii) $\frac{1}{5} = \frac{6}{\square}$

iii) $\frac{2}{3} = \frac{6}{\square}$

iv) $\frac{3}{13} = \frac{6}{\square}$

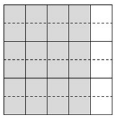
b) Write each of the fractions in part a) in order from smallest to greatest

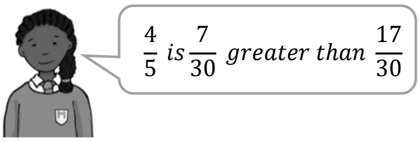
4.



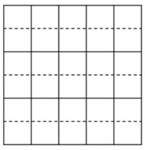
This diagram shows $\frac{17}{30}$. Use the diagrams to work out how much greater each fraction is than $\frac{17}{30}$.

e.g. $\frac{4}{5}$:



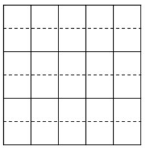


a)

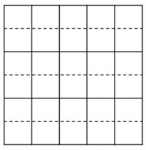


$\frac{3}{5}$

b) $\frac{5}{6}$



c) $\frac{2}{3}$



5. Use a similar method to decide which of the fractions is greater, and how much greater:

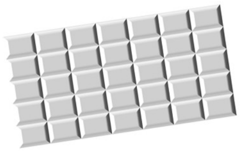
a) $\frac{4}{5}$ or $\frac{5}{7}$

b) $\frac{5}{6}$ or $\frac{9}{11}$

c) $\frac{3}{10}$ or $\frac{4}{15}$

d) $\frac{9}{8}$ or $\frac{23}{20}$

6. 3 friends share a chocolate bar. Polly ate $\frac{2}{5}$ of the chocolate bar, Kim ate $\frac{3}{7}$ of the chocolate bar, and Niels ate the rest.



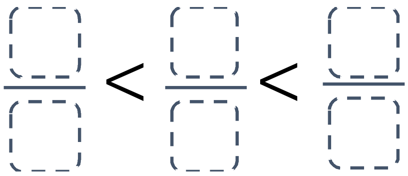
a) Who ate more, Polly or Kim?

b) What fraction did Niels eat?

Questions for depth:

1.

a) How many ways can you use the integers 1–6 to complete the following:



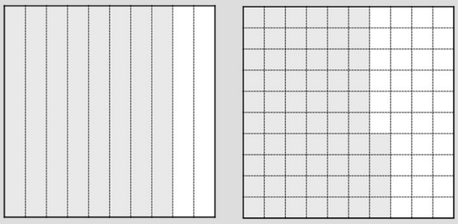
b) In which solution(s) is the smallest and largest fraction furthest apart?

Week 6 Session 3: Decimal fractions

Concept Corner

We can use ___________ notation to help us order fractions.

For example from the diagram we can see that _____ > ______



0.84	0.64
0.9	0.8
decimal	

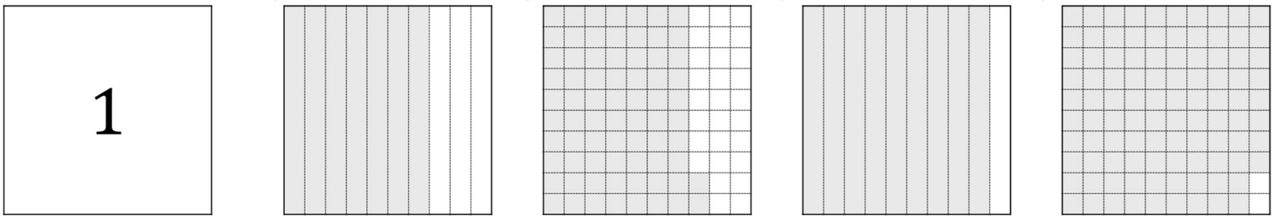
Writing fractions in decimal notation can help us to order.

e.g. $\frac{9}{10}=$ _____ and $\frac{21}{25}=$ _____ so $\frac{9}{10}>\frac{21}{25}$

1. For each diagram decide what **decimal** is represented by:

i) the shaded section
ii) the white section

a) b) c) d)

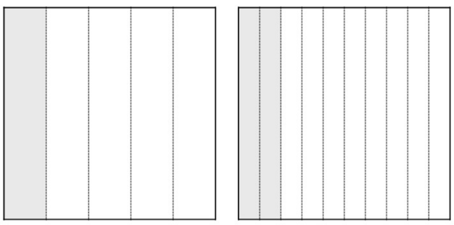


iii) Place the four decimal fractions in ascending order.

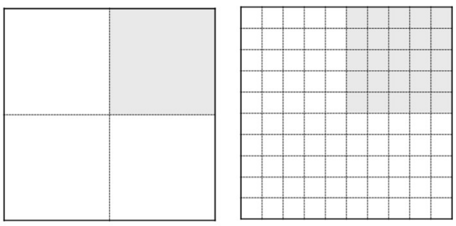
2. Copy and complete the equivalent statements for:

i) the shaded section
ii) the white section

a)



b)



$\frac{\square}{5}=\frac{\square}{10}=\square.\square$

$\frac{\square}{4}=\frac{\square}{100}=\square.\square\square$

3.

a) Copy and complete the following:

$$\frac{3}{10} = \square . \square \qquad \frac{31}{100} = \square . \square\square \qquad \frac{2}{5} = \frac{\square}{10} = \square . \square$$

$$\frac{17}{50} = \frac{\square}{100} = \square . \square\square \qquad \frac{8}{25} = \frac{\square}{100} = \square . \square\square$$

b) Write the fractions in **ascending** order

c) Find a fraction that lies between $\frac{31}{100}$ and $\frac{8}{25}$, write it in decimal form.

4. How many numbers can you create by placing the number cards, without repeats, in to the spaces? Record them in **ascending** order.

0 1 2

$$\square . \square\square$$

5. How many different fractions can you create by selecting two of the cards to be the numerator and denominator of the fraction?

   Write each fraction in **decimal notation** and record them in ascending order.

4 5 2 10

$$\frac{\square}{\square}$$

Questions for depth:

1. How many was can you complete the following, with no repeats?

0 1 2 3 4

$$\frac{1}{\square} < \square . \square\square < \frac{1}{\square}$$

Week 6 Session 4: Mixed comparisons

Concept Corner

Depending on the situation, different ____________ for ____________ fractions can be more or less useful.

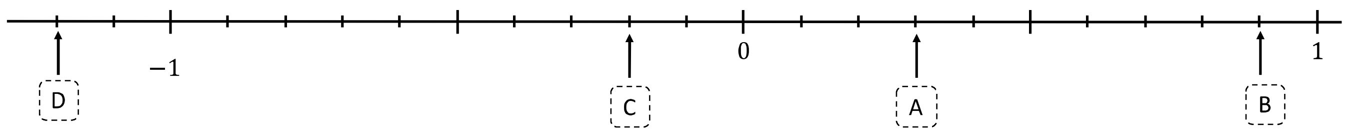
For example you could deduce that:

$\frac{1}{3}$ ____ $\frac{1}{5}$ by comparing their ____________.

$\frac{17}{50}$ ____ 0.32 by noting that $\frac{17}{50}$ = ______

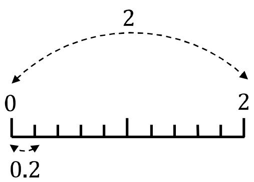
<	denominators
>	ordering
0.34	strategies

1. What decimals are shown on the numberline?



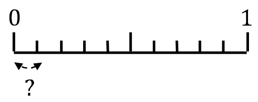
2. Find the size of the interval in each of the number lines:

For example…

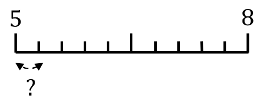


$\frac{2}{10} = 0.2$

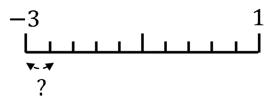
a)



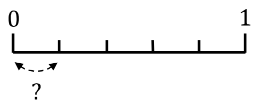
b)



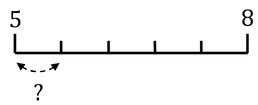
c)



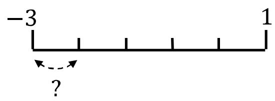
d)



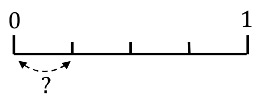
e)



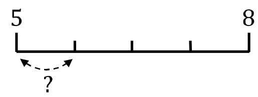
f)



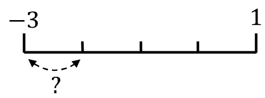
g)



h)



i)



3. Draw your own numberline, with an appropriate scale, indicating:

0.3 , 0.7 , −1.5 and 2.3.

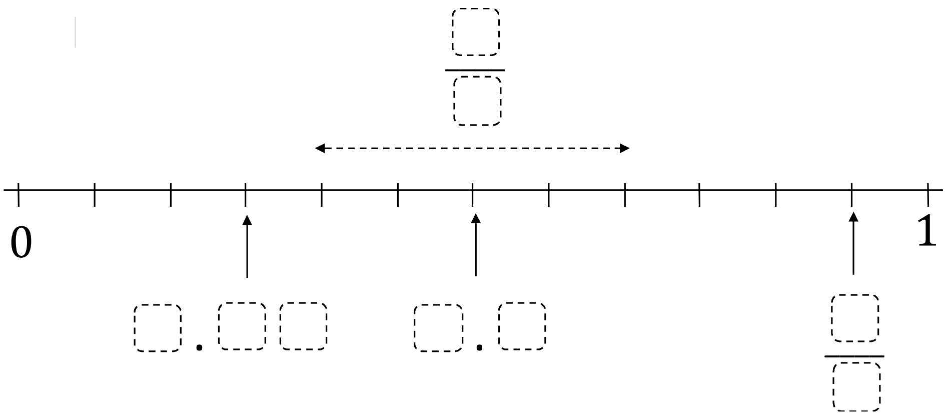
4. Copy and complete the following, use the correct symbol >, < or = .

a) $\frac{3}{4} \ \square \ \frac{1}{4}$ b) $\frac{1}{5} \ \square \ \frac{1}{4}$ c) $\frac{3}{4} \ \square \ \frac{2}{3}$

d) $0.7 \ \square \ \frac{3}{4}$ e) $0.6 \ \square \ \frac{3}{8}$ f) $0.7 \ \square \ \frac{7}{11}$

g) $0.25 \ \square \ \frac{6}{25}$ h) $0.23 \ \square \ \frac{1}{4}$ i) $0.3 \ \square \ \frac{1}{3}$

5. Copy and complete the numberline:



6. How many was can you insert two of the cards in to the spaces to make the statements true:

1 2 3 4 5 6 7 8 9

a) $\frac{1}{\square} < \frac{1}{\square}$ b) $0.\square < \frac{1}{\square}$ c) $\frac{1}{\square} < 0.\square$

Questions for depth:

1. Charlie thinks that you can find a fraction that lies in between any pair of numbers. Do you agree? Explain your reasoning.

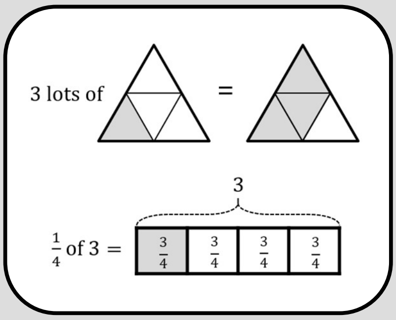
Week 7: Fractions 3

Session 1: Modelling multiplication I

Concept Corner

When ___________ fractions and integers, it is useful to use models to investigate the calculation.

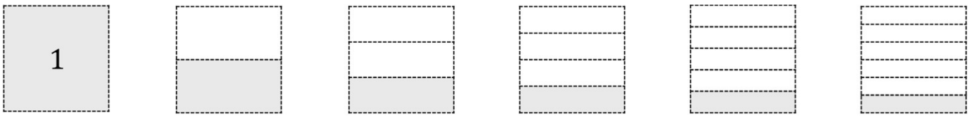
If we want to multiply 3 and $\frac{1}{4}$ we can consider 3 ___________ $\frac{1}{4}$. Alternatively, we can consider $\frac{1}{4}$ of 3. We know this will give the same answer because of the _______________ property of multiplication.



- lots of
- multiplying
- commutative

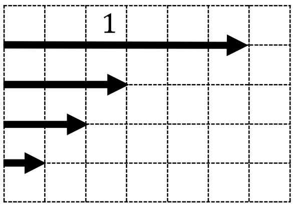
1. What unit fraction is represented?

a)



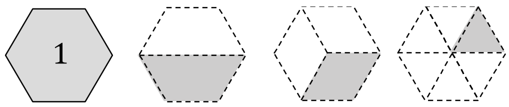
e.g. $\frac{1}{2}$ ___ ___ ___ ___

b)



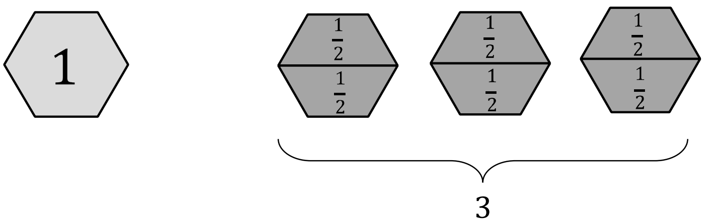
___
___
___

c)



___ ___ ___

2. Tom drew a diagram that represents: $6 \times \frac{1}{2} = 3$



a) Draw your own diagram to represent:

i) $6 \times \frac{1}{3}$

ii) $8 \times \frac{1}{4}$

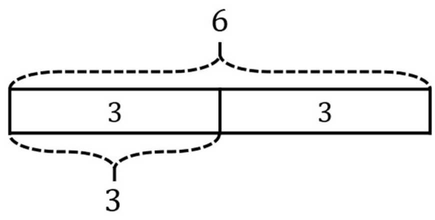
iii) $3 \times \frac{1}{4}$

iv) $4 \times \frac{1}{3}$

b) Work out the value of each calculation.

c) Write a word problem for each of the calculations

3. Andi drew a different model to represent $\frac{1}{2} \times 6 = 3$



a) Draw a similar diagram to represent:

i) $\frac{1}{3} \times 12$

ii) $\frac{1}{5} \times 35$

iii) $\frac{1}{2} \times 3$

iv) $\frac{1}{4} \times -20$

b) Work out the value of each calculation.

4. Copy and complete the calculations:

a) $\frac{1}{3} \times 12 = 12 \times$ __ = __

b) $\frac{1}{3} \times$ ___ = ___ $\times \frac{1}{3} = 8$

c) $\frac{1}{6} \times 12 = 12 \times$ __ = __

d) $\frac{1}{6} \times$ ___ = ___ $\times \frac{1}{6} = 4$

e) $\frac{1}{6} \times 18 =$ ___ ÷ __ = __

f) $7 \times \frac{1}{5} =$ ___ $\div 5 = \frac{?}{?}$

g) $\frac{1}{7} \times n =$ __ $\div 7 =$ ___

h) $n \times \frac{1}{m} =$ ___ ÷ ___ $= \frac{?}{?}$

5. Write a word problem for each calculation in Q5.

6. Find **ten pairs** of numbers whose product is 1.

Questions for depth:

1. Plot each of the pairs in Q7 as a coordinate on an x and y axis.
   e.g. $\frac{1}{2} \times 2 = 1$ plot the pair $(\frac{1}{2}, 2)$

2.

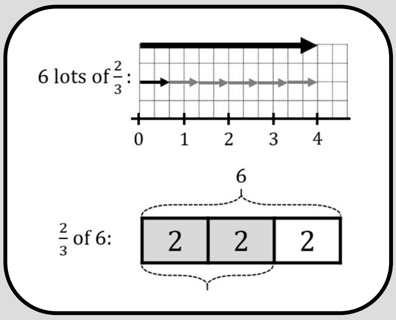
a) Find the value of: $\left(\frac{1}{6} \times 5\right) \times \left(8 \times \frac{1}{5}\right) \times \left(6 \times \frac{1}{8}\right)$

b) Find similar **products** that have the same value.

Week 7 Session 2: Modelling multiplication II

Concept Corner

We can apply the same models to multiplying non-unit fractions. In this example, the ___________ of $\frac{2}{3}$ and 6 can be found by working out 6 lots of $\frac{2}{3}$, or by working out $\frac{2}{3}$ of _____.



4

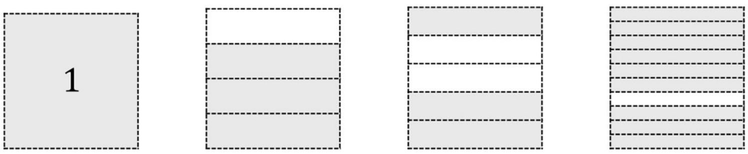
product

6

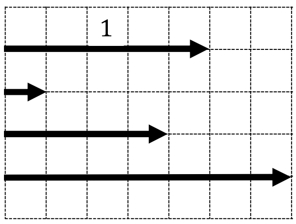
These models also apply to percentages, e.g. $30\% \times 40 = \frac{30}{100}$ of $40 = 40$ lots of $\frac{30}{100}$

1. What number is represented? Write your answer as a fraction, decimal and percentage.

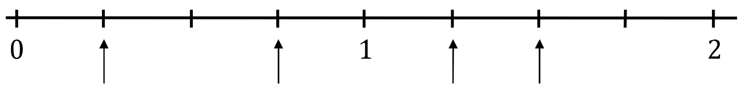
a)



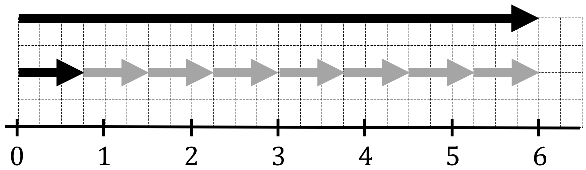
b)



c)



2. Tom drew a diagram that represents: $8 \times \frac{3}{4} = 6$



a) Draw your own diagram to represent:

i) $3 \times \frac{2}{3}$ ii) $4 \times \frac{3}{4}$

iii) $9 \times \frac{2}{3}$ iv) $5 \times \frac{2}{3}$

b) Work out the value of each calculation.

c) Write a word problem linked to each calculation.

Mathematics Mastery

Ark

3. Copy and complete the following:

a) $\frac{1}{4} \times 12 = 12 \div \_\_ = \_\_$ b) $\frac{3}{4} \times 12 = 12 \div \_\_ \times \_\_ = \_\_$ c) $\frac{5}{4} \times 12 = 12 \div \_\_ \times \_\_ = \_\_$

d) $2 \times \frac{1}{5} = 2 \div \_\_ =$ e) $2 \times \frac{4}{5} = \_\_ \div \_\_ \times 4 = \_\_$ f) $2 \times \frac{7}{5} = \_\_$

4. Write each of the calculations in Q3 as decimal multiplications.

e.g. $\frac{1}{4} \times 12 = 0.25 \times 12$

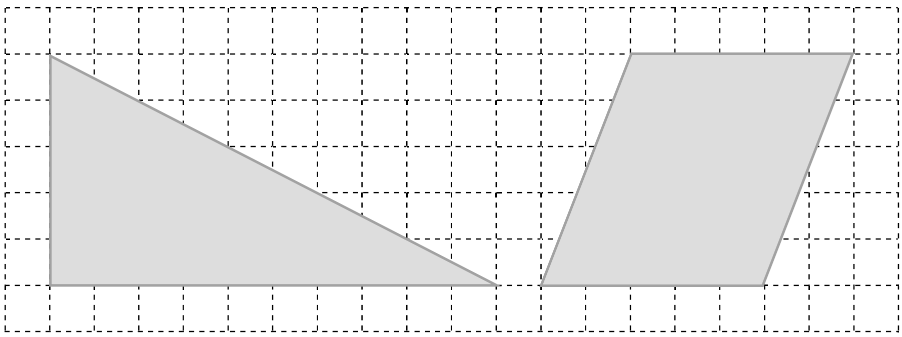
5. Copy and complete the following:

a) $0.3 = \frac{?}{10} = \frac{?}{100}$ b) $0.25 = \frac{?}{4} = \frac{?}{100}$ c) $0.75 = \frac{?}{4} = \frac{?}{100}$

d) $0.3 \times 70 = \_\_$ e) $44 \times 0.25 = \_\_$ f) 0.75×28

g) 30% of 12 h) 25% of 44 i) 75% of 28

6. Draw enlarged versions of the shapes below by a scale factor of:
   a) 0.6 b) $\frac{2}{5}$ c) 0.8



Questions for depth:

1. A monkey starts with 75 bananas.
   - He eats $\frac{1}{3}$ of them and throws one away.
   - He then eats $\frac{4}{7}$ of what is left and throws one away.
   - He then eats $\frac{9}{10}$ of what is left and throws one away.

   a) How many does he have left over?
   b) How can you change the three fractions and end up with the same amount?

Week 7 Session 3: Multiplying fractions I

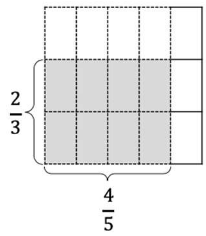
Concept Corner

We can use an ____________ to help us multiply two fractions. This diagram shows a that the product of $\frac{2}{3}$ and $\frac{4}{5}$ is _____ .

The ________________ is represented by the shaded area, and is a rectangle with an 2×4 parts.

The ________________ is represented by entire square, and is divided into 3×5 parts.

This can be thought of as "2 thirds of 4 fifths" or "4 fifths of 2 thirds".



$\frac{8}{15}$

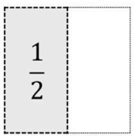
denominator

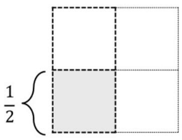
numerator

array

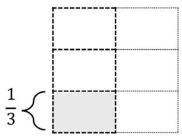
1. Use the diagram to copy and complete the calculations:

a)

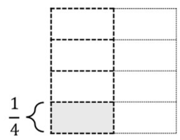




$\frac{1}{2}$ of $\frac{1}{2}$ is ____ $\quad \frac{1}{2} \times \frac{1}{2} =$ ____

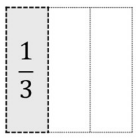


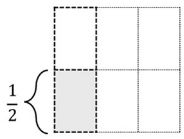
$\frac{1}{3}$ of $\frac{1}{2}$ is ____ $\quad \frac{1}{3} \times \frac{1}{2} =$ ____



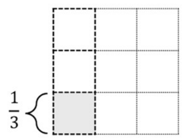
$\frac{1}{4}$ of $\frac{1}{2}$ is ____ $\quad \frac{1}{4} \times \frac{1}{2} =$ ____

b)

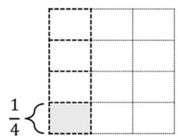




$\frac{1}{2}$ of $\frac{1}{3}$ is ____ $\quad \frac{1}{2} \times \frac{1}{3} =$ ____



$\frac{1}{3}$ of $\frac{1}{3}$ is ____ $\quad \frac{1}{3} \times \frac{1}{3} =$ ____



$\frac{1}{4}$ of $\frac{1}{3}$ is ____ $\quad \frac{1}{4} \times \frac{1}{3} =$ ____

2. Draw your own diagrams, to illustrate the following calculations:

a) $\frac{1}{2} \times \frac{1}{5}$

b) $\frac{1}{3} \times \frac{1}{5}$

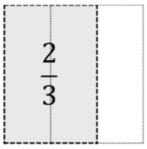
c) $\frac{1}{4} \times \frac{1}{5}$

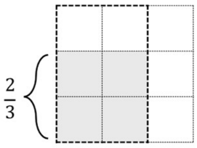
d) $\frac{1}{5} \times \frac{1}{2} =$

e) $\frac{1}{5} \times \frac{1}{3}$

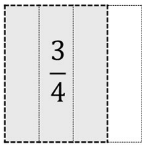
f) $\frac{1}{5} \times \frac{1}{4}$

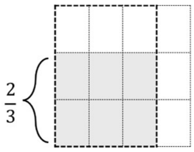
3. Use the diagrams to copy and complete the calculations:



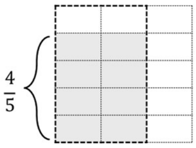


$\frac{2}{3}$ of $\frac{2}{3}$ is ___ $\frac{2}{3} \times \frac{2}{3} =$ ___

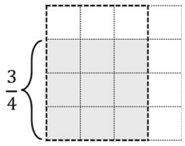




$\frac{2}{3}$ of $\frac{3}{4}$ is ___ $\frac{2}{3} \times \frac{3}{4} =$ ___



$\frac{4}{5}$ of $\frac{2}{3}$ is ___ $\frac{4}{5} \times \frac{2}{3} =$ ___



$\frac{2}{3}$ of $\frac{2}{3}$ is ___ $\frac{2}{3} \times \frac{2}{3} =$ ___

4. Calculate the following, simplify your answers:

a) $\frac{1}{3} \times \frac{1}{5}$ b) $\frac{2}{3} \times \frac{1}{5}$ c) $\frac{3}{3} \times \frac{1}{5}$

d) $\frac{2}{3} \times \frac{4}{5}$ e) $\frac{2}{3} \times \frac{5}{5}$ f) $\frac{2}{3} \times \frac{6}{5}$

5. Find five pairs of fractions whose product is …

a) $\frac{1}{2}$ b) 1 c) $\frac{7 \times 5}{6 \times 8}$

6. Copy and the complete the following:

a) $\frac{3}{5} \times \frac{5}{7} = \frac{\square \times \square}{5 \times 7} = \frac{\square}{5} \times \frac{3}{7} = \square \times \frac{3}{7} = \frac{\square}{\square}$

b) $\frac{3}{5} \times \frac{10}{7} = \frac{\square \times \square}{5 \times 7} = \frac{\square}{5} \times \frac{3}{7} = \square \times \frac{3}{7} = \frac{\square}{\square}$

Questions for depth:

1. Place the following in ascending order:

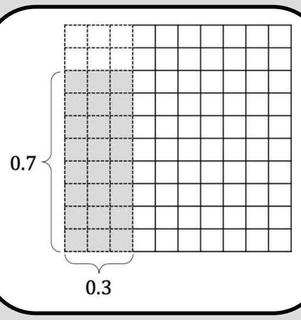
a) $\left(\frac{2}{3}\right)^2$ b) $\left(\frac{2}{3}\right)^3$ c) $\frac{2}{3} \times \frac{4}{5}$ d) $\left(\frac{2}{3}\right)^2 \times \frac{4}{5}$ e) $\left(\frac{4}{5}\right)^3$

2. Does squaring a number increase its value? Explain your answer.

Week 7 Session 4: Multiplying fractions II

Concept Corner

By converting ____________ to fractions, we can use the same model to find the product of decimals.



decimals

21 10

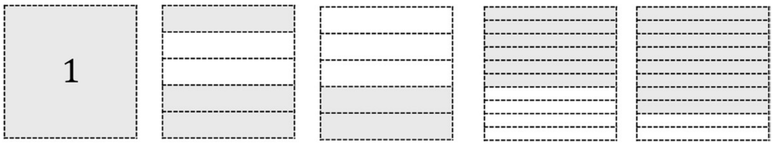
3

E.g.

$$0.7 \times 0.3 = \frac{7}{\square} \times \frac{\square}{10} = \frac{7 \times 3}{10 \times 10} = \frac{\square}{100} = 0.21$$

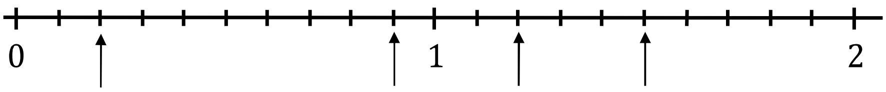
1. What number is represented? Write your answer as a fraction and a decimal.

a)



___ ___ ___ ___

b)



___ ___ ___ ___

2. Copy and complete the following:

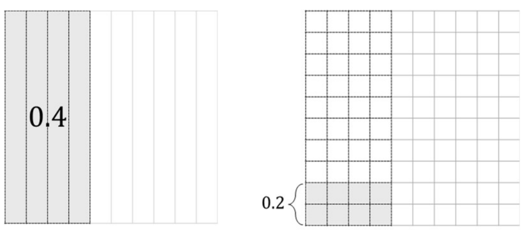
a) $0.4 \times 0.6 = \frac{\square}{10} \times \frac{\square}{10} = \frac{\square}{100} = \square.\square\square$

b) $0.2 \times 0.9 = \frac{\square}{10} \times \frac{\square}{10} = \frac{\square}{100} = \square.\square\square$

c) $0.2 \times 1.5 = \frac{\square}{5} \times \frac{\square}{2} = \frac{\square}{10} = \square.\square$

d) $1.2 \times 1.5 = \frac{\square}{5} \times \frac{\square}{2} = \frac{\square}{10} = \square.\square$

3. Draw a model to illustrate two of the calculations in Q2. For example:



$$0.2 \times 0.4 = \frac{2}{10} \times \frac{4}{10} = \frac{8}{100} = 0.08$$

Mathematics Mastery

Ark

4. Copy and complete the following:

a) $2.5 \times 0.2 = 2.5 \times \frac{\square}{5} = 2.5 \div 5 = \square . \square$

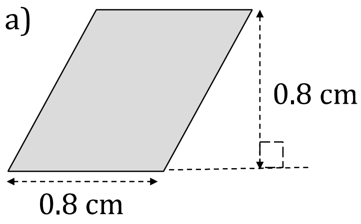
b) $2.5 \times 0.2 = \frac{\square}{2} \times \frac{\square}{5} = \frac{\square}{\square} = \square . \square$

c) $2.5 \times 0.2 = 2.5 \times \frac{\square}{10} = 2.5 \times \square \div \square = \square . \square$

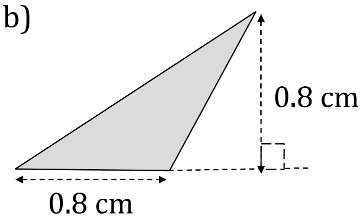
d) $2.5 \times 0.2 = 2 \times \square + 0.5 \times \square = \square . \square + \square . \square$

5. Find the area of each shape:

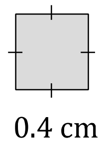
a)



b)



c)



6. **Sketch** different shapes that have an area of 1 cm^2.

7. Calculate the value of each expression when:

ab $\quad b^2$ $\quad a^2$ $\quad a^3$ $\quad b^3$

a) $a = 0.1$ and $b = 0.2$

b) $a = 0.2$ and $b = 0.4$

c) $a = -0.2$ and $b = 0.4$

d) $a = -0.2$ and $b = -0.4$

Questions for depth:

1. **Sketch** and label an enlargement of the shapes in Q5 using a scale factor of:

a) 0.5 b) 0.25 c) 0.1 d) 0.8

Week 8: Fractions 4

Session 1: Dividing fractions by integers

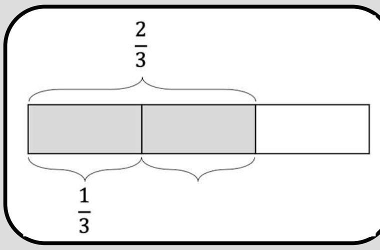
Concept Corner

Dividing fractions by integers might be thought of different ways. For example, $\frac{2}{3} \div 2$ can be thought of as:

$\frac{2}{3}$ split into 2 ______________ groups is…

or:

$\frac{2}{3}$ is 2 of what?

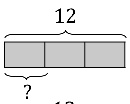


$\frac{1}{3}$ | equal

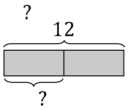
Using the model, we can see that the answer to both questions is ________.

We can also think rewrite this as $\frac{2}{3} \times \frac{1}{2}$ as use multiplication models.

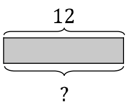
1. Copy and complete the following:

a) 

"Twelve is three lots of ____" $12 \div 3 = \square$

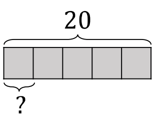
b) 

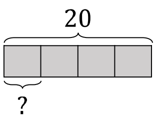
"Twelve is two lots of ____" $12 \div 2 = \square$

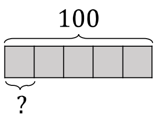
c) 

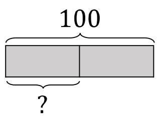
"Twelve is one lot of ____" $12 \div 1 = \square$

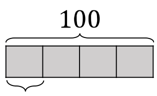
2. Find the missing value and write the division:

a) 

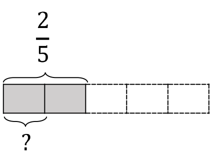
b) 

c) 

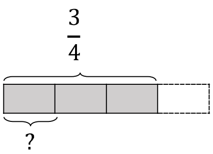
d) 

e) 

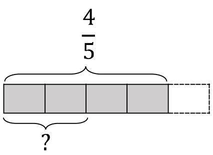
3. Work out the division calculation linked to the bar model:

e.g. 

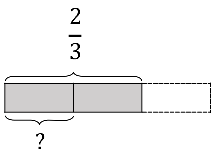
$\frac{2}{5} \div 2 = \frac{1}{5}$

a) 

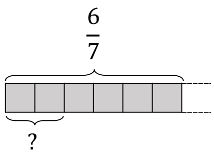
$\frac{3}{4} \div 3 = \frac{\square}{\square}$

b) 

$\frac{4}{5} \div 2 = \frac{\square}{\square}$

c) 

$\frac{2}{3} \div \square = \frac{\square}{\square}$

d) 

$\frac{6}{7} \div \square = \frac{\square}{\square}$

4. Draw a bar model and compute the division:

   a) $\frac{3}{4} \div 3$ b) $\frac{2}{3} \div 2$ c) $\frac{4}{3} \div 4$

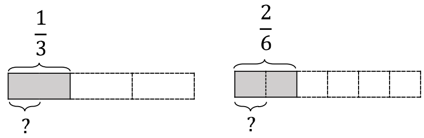
   d) $\frac{4}{7} \div 2$ e) $\frac{6}{7} \div 2$ f) $\frac{6}{5} \div 3$

5. Compute the following

   a) $\frac{1}{3} \div 3$ b) $\frac{1}{4} \div 2$ c) $\frac{1}{2} \div 3$

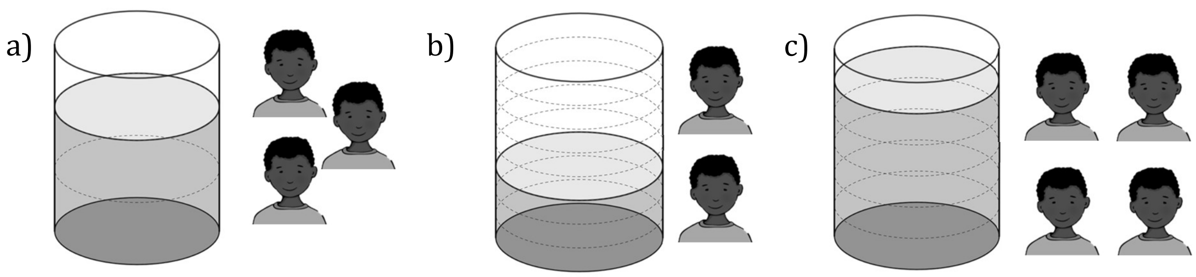
   d) $\frac{2}{3} \div 3$ e) $\frac{3}{4} \div 2$ f) $\frac{5}{2} \div 3$

e.g.



$\frac{1}{3} \div 2 = \frac{2}{6} \div 2 = \frac{1}{6}$

6. Each cylinder holds 1 L. If shared equally, how much soda does each person receive?



In which situation do the people receive the most soda?

7. Copy and complete the following:

a) $20 \div 2 = 20 \times \frac{\square}{\square}$ b) $\frac{4}{5} \div 2 = \frac{4}{5} \times \frac{\square}{\square}$ c) $\frac{3}{8} \div 4 = \frac{3}{8} \times \frac{\square}{\square}$

Write each of the divisions in Q5 as a product of two numbers.

Questions for depth:

1.

a) A group of people share $1\frac{2}{3}$ L of soda, another group shares $2\frac{5}{6}$ L . Each person receives the same amount of soda, how many people could be in each group?

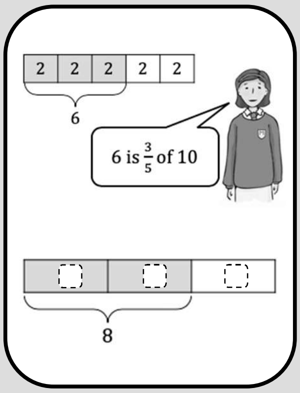
b) Change the amount of soda each group shares, can you always find a solution?

Session 2: Modelling division by fractions I

Concept Corner

When ___________ a number by a fraction, such as $8 \div \frac{2}{3}$, we can consider that 8 is $\frac{2}{3}$ of the result.

Therefore, $8 \div \frac{2}{3} =$ _______



4 | 12 | dividing | 4 | 4

1. Write a corresponding division for each sentence:

e.g. Twelve is three lots of ________. $12 \div 3 =?$

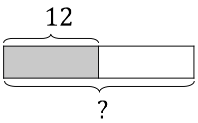
a) Ten is five lots of ________.

b) Thirty is ten lots of ________.

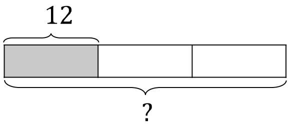
c) Twenty is four groups of ________.

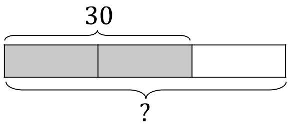
d) Eighty is ________ scaled by a factor of 20.

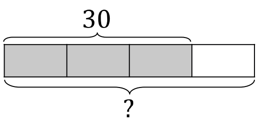
2. For each bar model write a matching sentence and division:

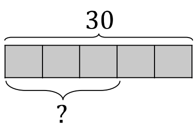
e.g. 

" Twelve is half of __________ " $12 \div \frac{1}{2} = 24$

a) 

b) 

c) 

d) 

3. Calculate the following and draw your own bar model:

a) $8 \div \frac{1}{5}$

b) $8 \div \frac{2}{5}$

c) $8 \div \frac{4}{5}$

d) $9 \div \frac{3}{4} =$ ___

e) $9 \div \frac{3}{5} =$ ___

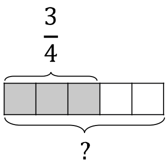
f) $9 \div \frac{3}{2}$

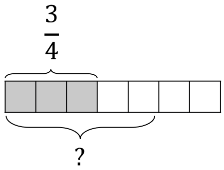
g) $18 \div \frac{3}{4} =$ ___

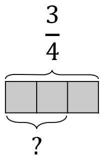
h) $27 \div \frac{3}{5} =$

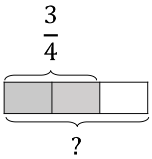
i) $90 \div \frac{3}{2}$

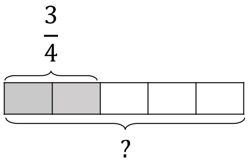
4. Find the missing values for each of the bar models

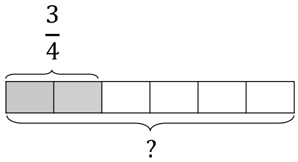
a) 

b) 

c) 

d) 

e) 

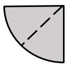
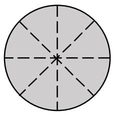
f) 

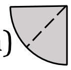
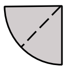
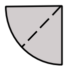
For each bar model write a matching division and multiplication

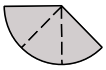
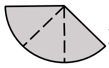
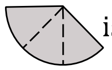
e.g. for a)

$\frac{3}{4} \div \frac{3}{5} = ?$ and $\frac{3}{4} = \frac{3}{5} \times ?$

5. Sketch a diagram to complete the statement

e.g.  is $\frac{1}{4}$ of 

a)  is $\frac{1}{2}$ of ____________ b)  is $\frac{1}{3}$ of ____________ c)  is $\frac{2}{3}$ of ____________

d)  is $\frac{1}{2}$ of __________ e)  is $\frac{3}{4}$ of __________ f)  is $\frac{1}{3}$ of __________

For each diagram write a matching division and multiplication

6. Calculate the following and draw your own diagram

a) $\frac{3}{5} \div \frac{3}{4}$ b) $\frac{6}{7} \div \frac{3}{4}$ c) $\frac{6}{5} \div \frac{3}{4}$

d) $\frac{4}{3} \div \frac{2}{5}$ e) $1\frac{1}{3} \div \frac{4}{5}$ f) $\frac{3}{4} \div \frac{2}{5}$

Questions for depth:

1. Write equivalent expressions for each of the following:

a) $x \div \frac{1}{5}$ b) $x \div \frac{1}{3}$ c) $x \div \frac{1}{6}$

d) $3n \div \frac{3}{4}$ e) $3n \div \frac{3}{5}$ f) $3n \div \frac{3}{7}$

2. Create your own word problems that match the division calculations in Q6.

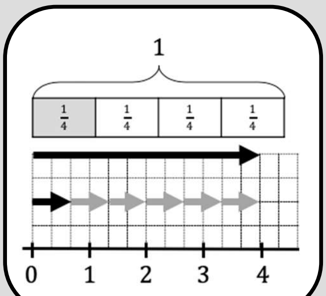
Session 3: Modelling division by fractions II

Concept Corner

Another way to think about __________ fractions is to consider how many times the _________ *goes in* to the dividend.

E.g. $1 \div \frac{1}{4}$: how many times does $\frac{1}{4}$ go in to 1? _____

Using the same technique we can work out that 4 ÷ ____ = 6



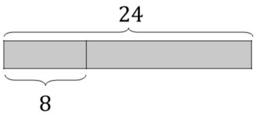
dividing

divisor

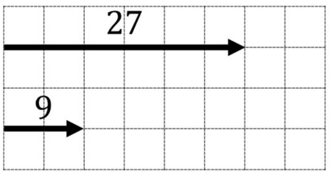
$\frac{2}{3}$

4

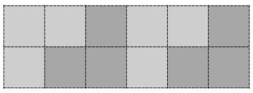
1. What value is missing each of the following:



"Eight goes into twenty-four _____ times" 24 ÷ 8 = ⬚



"9 scaled by a factor of ___ is 27" 27 ÷ 9 = ⬚

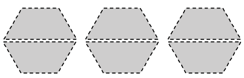


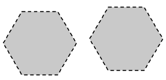
"Three goes in to twelve ___ times" 12 ÷ 3 = ⬚

2. Draw similar diagrams for the following:

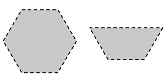
a) 6 ÷ 2 b) 42 ÷ 6 c) 24 ÷ 8

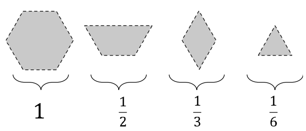
3. Use the diagrams to calculate the following:

a)  is ⬚ lots of  $3 \div \frac{1}{2} =$ ⬚

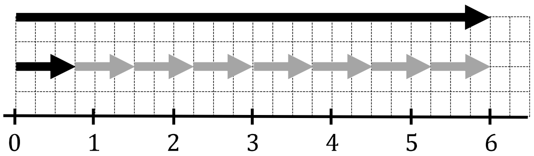
b)  is ⬚ lots of  $2 \div \frac{1}{3} =$ ⬚

c)  is ⬚ lots of  $\frac{1}{2} \div \frac{1}{6} =$ ⬚

d)  is ⬚ lots of  $1\frac{1}{2} \div \frac{1}{6} =$ ⬚



4. Tom drew a diagram that represents: $6 \div \frac{3}{4}$



a) Draw a similar diagram to represent:

i) $2 \div \frac{1}{3}$ ii) $2 \div \frac{2}{3}$

iii) $1 \div \frac{1}{4}$ iv) $3 \div \frac{1}{4}$

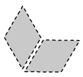
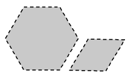
b) Work out the value of each calculation.

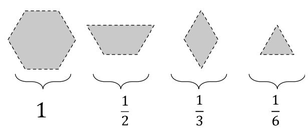
5. Copy and complete the following:

a)  is $\frac{\square}{\square}$ of  $\frac{1}{3} \div \frac{1}{2} = \frac{\square}{\square}$

b)  is $\frac{\square}{\square}$ of  $\frac{1}{2} \div \frac{1}{3} = \frac{\square}{\square}$

c)  is $\frac{\square}{\square}$ of  $\frac{1}{6} \div \frac{1}{2} = \frac{\square}{\square}$

d)  is $\frac{\square}{\square}$ of  $\frac{2}{3} \div 1\frac{1}{3} = \frac{\square}{\square}$



6. Calculate the following:

a) $\frac{3}{5} \div \frac{1}{5}$ b) $\frac{3}{5} \div \frac{1}{10}$ c) $\frac{6}{10} \div \frac{3}{10}$ d) $\frac{3}{5} \div \frac{3}{10}$

e) $\frac{6}{15} \div \frac{2}{15}$ f) $\frac{2}{5} \div \frac{2}{15}$ g) $\frac{3}{5} \div \frac{4}{5}$ h) $\frac{6}{10} \div \frac{12}{15}$

Questions for depth:

1. If $\frac{3}{7}$ L of paint can cover 1 m^2 . Complete the following:

a) 3 L of paint can cover ______

b) 1 L of paint can cover ______

c) $\frac{3}{4}$ L of paint can cover ______

d) n L of paint can cover ______

Session 4: Dividing with fraction in mixed contexts

Concept Corner

We can use fraction _________ to help us solve problems in different contexts.

6 | division | 9

For example:

Charlie has lots of eggs and flour, but only 6 bags of sugar.

It takes $\frac{2}{3}$ of a bag of sugar to bake a cake.

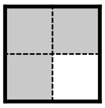
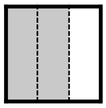
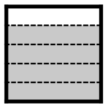
How many cakes can he bake?

How many times does $\frac{2}{3}$ go into ___?

$6 \div \frac{2}{3} =$ ___

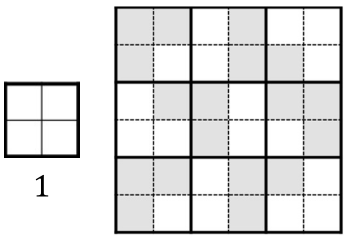
He can bake 9 cakes.

1. Decide what fraction of the square is shaded:

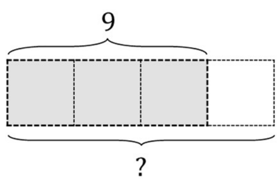
a)  b)  c) 

Represent the three fractions above in a different way.

2. Harry has drawn two diagrams to illustrate: $9 \div \frac{3}{4} = 12$



"Three-quarters goes in to nine, **twelve** times"



"Nine is three-quarters of **twelve**"

Draw two similar diagrams to illustrate:

a) $3 \div \frac{3}{4}$ b) $4 \div \frac{2}{3}$ c) $4 \div \frac{4}{5}$

3. Draw five rectangles that have an area of $10\ \text{cm}^2$:

$10\ \text{cm}^2$

4. Solve each problem and write the corresponding division:

a) It takes Tom $\frac{1}{4}$ of an hour to ice a cake. How many can he ice in $2\frac{3}{4}$ hours?

b) Siobhan has mown $\frac{2}{3}$ of the lawn in the garden. She has mown $20\ \text{m}^2$ so far. What is the area of the lawn?

c) 1 L of paint covers $\frac{3}{4}$ of a wall, how many litres are needed to cover the wall?

5. Calculate the following:

a) $\frac{2}{3} \div \frac{1}{3}$ b) $\frac{2}{3} \div \frac{1}{6}$ c) $\frac{2}{3} \div \frac{1}{2}$ d) $\frac{2}{3} \div \frac{1}{4}$

e) $\frac{1}{4} \div \frac{1}{4}$ f) $\frac{1}{4} \div \frac{1}{2}$ g) $\frac{1}{4} \div \frac{1}{6}$ h) $\frac{1}{4} \div \frac{1}{3}$

6. Write two word problems matching the calculations in Q2:

e.g. For $9 \div \frac{3}{4} = 12$:

1. *"Harry has £9. If oranges cost 75 p each, how many can he buy?"*
2. *"Harry has painted three quarters of a wall. He has painted 9* m^2 *in total, what is the area of the wall?"*

Questions for depth:

1. Using each of the number cards 2 3 4 6, how many ways can you complete:

a) $\frac{\square}{\square} \div \frac{\square}{\square} < 1$ b) $\frac{\square}{\square} \div \frac{\square}{\square} = 1$ c) $\frac{\square}{\square} \div \frac{\square}{\square} > 1$

Week 9: Fractions 5

Session 1: Adding and subtracting fractions

Concept Corner

When fractions are written with a ___________ denominator, we can simply add or _____________ how many parts there are.

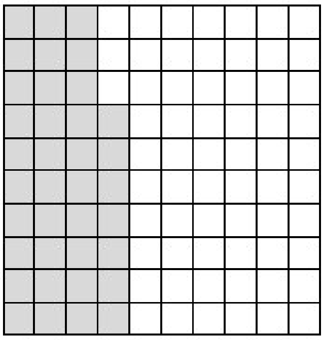
E.g. _____ sevenths add 3 sevenths = 8 sevenths

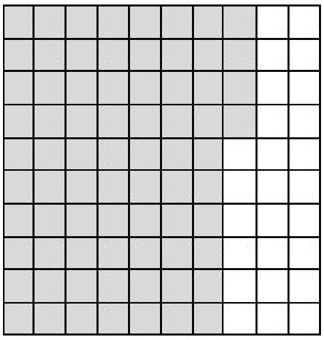
$\frac{4}{5} - \frac{3}{5} = \frac{\square}{5}$

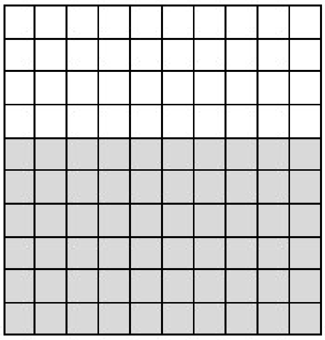
3 tenths + 1 tenths = 0.3 + _____ = $\frac{3}{10} + \frac{1}{10} = \frac{4}{10}$

0.1	subtract
5	common

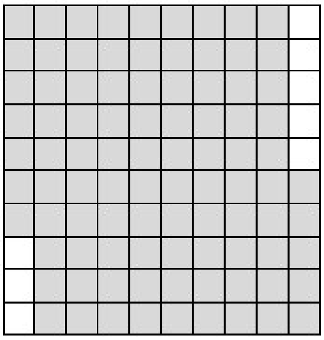
1. In the images below, the whole hundred square represents 1.
   What are the values of the shaded areas? Write your answers as fractions and decimals. An example has been done for you.

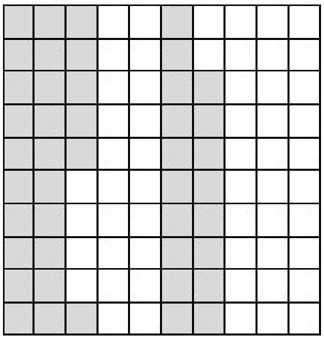
i) 

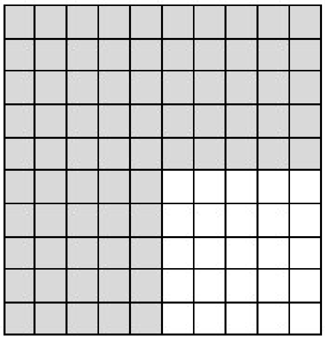
ii) 

iii) 

0.35 $\frac{35}{100}$

iv) 

v) 

vi) 

a) Draw your own hundred squares and shade them to represent:

i) 0.8 iii) 0.07 iii) $\frac{13}{100}$

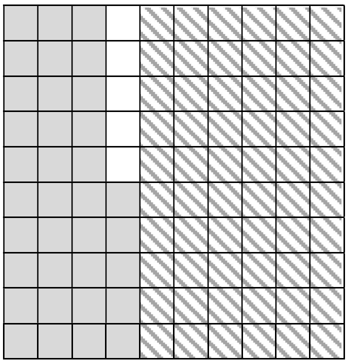
b) Is it possible to represent all of the numbers in part a) **together** on a single hundred square? Explain your answer.

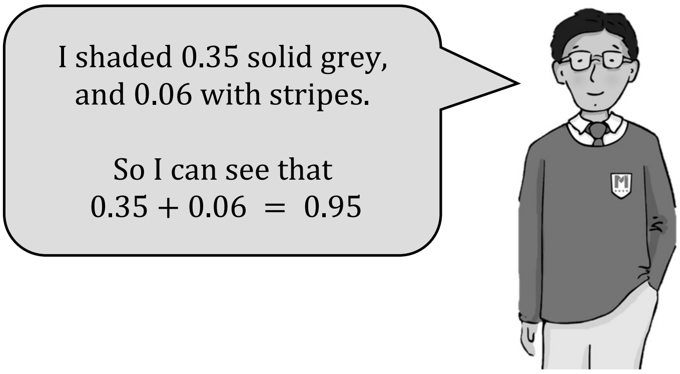
2. Copy and complete the addition calculations below.

a) $\frac{2}{10}+\frac{\quad}{10}=\frac{7}{10}$ b) $\frac{3}{8}-\frac{2}{8}=\frac{\quad}{8}$ c) $\frac{7}{15}+\frac{8}{15}=$

d) $\frac{24}{25}=\frac{\quad}{25}+\frac{\quad}{25}$ e) $\frac{3}{7}+\frac{\quad}{7}=1\frac{2}{7}$ f) $1\frac{3}{4}=\frac{\quad}{4}-\frac{\quad}{4}$

3. Phil is using a hundred square to represent 1. Look at his statement below.





a) Explain why Phil is **wrong**.

b) Draw your own hundred square and shade it to correctly represent 0.35 + 0.06.

4. Copy and complete the addition calculations below.

a) $\frac{2}{10}+\frac{1}{10}=\frac{\quad}{10}$ b) $\frac{20}{100}-\frac{15}{100}=\frac{\quad}{100}$ c) $\frac{25}{100}+\frac{15}{100}=$

d) $\frac{42}{100}-\frac{25}{100}=\frac{\quad}{100}$ e) $\frac{3}{10}+\frac{17}{100}=\frac{\quad}{100}$ f) $0.32+\quad\quad=0.47$

g) $0.57-\frac{\quad}{10}=0.47$ h) $\quad\quad=0.57-\frac{4}{100}$ i) $\frac{57}{100}+0.44=$

Questions for depth:

1. Look at the pattern of growing addition calculations below. If the pattern continues, will there ever be an addition with sum 1?

$$\frac{1}{100} \longrightarrow \frac{1}{100}+\frac{2}{100} \longrightarrow \frac{1}{100}+\frac{2}{100}+\frac{3}{100} \longrightarrow \frac{1}{100}+\frac{2}{100}+\frac{3}{100}+\frac{4}{100}$$

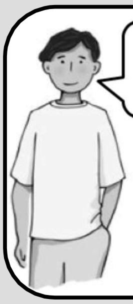
2. What values of n will give a pattern where a sum of 1 will occur?

$$\frac{1}{n} \longrightarrow \frac{1}{n}+\frac{2}{n} \longrightarrow \frac{1}{n}+\frac{2}{n}+\frac{3}{n} \longrightarrow \frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\frac{4}{n}$$

Session 2: Different denominators

Concept Corner

Using our understanding of ____________, we can make sensible statements about addition of fractions.



The _____ of these fractions will be less than 1 because both fractions are less than $\frac{1}{2}$

$$\frac{3}{7}+\frac{1}{3}$$

sum

addition

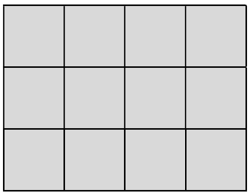
number

It is helpful to draw diagrams such as ______________ lines to make sense of the size of fractions and their sums.

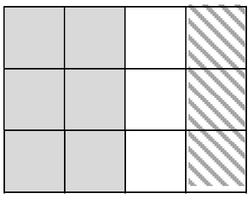
1. Put the fractions below into ascending order.

$\frac{2}{3}$ $\frac{2}{5}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{5}$

2. In the diagrams below each rectangular grid represents 1.
   The grids have been shaded to show fraction additions.
   Write out the addition each grid shows, an example has been done for you.

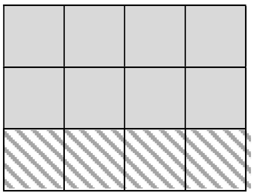


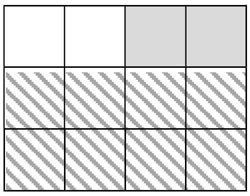
$= 1$

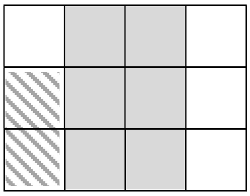


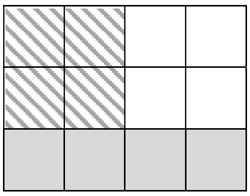
Example:

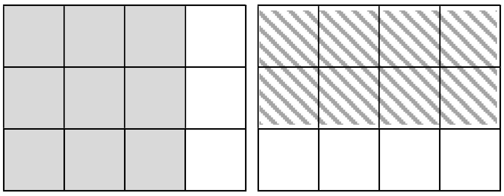
$\frac{1}{2}+\frac{1}{4}$

a) 

b) 

c) 

d) 

e) 

3. Look at your answers to question 2. Which of the additions you wrote in question 2 have an answer that is **less than 1**? Explain how you know.

4. Decide if the inequalities are **true or false** and explain how you know.

a) $1 > \frac{1}{2} + \frac{1}{4}$

b) $\frac{2}{3} + \frac{1}{2} > 1$

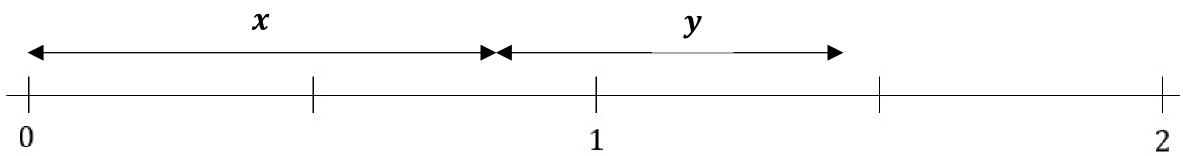
c) $\frac{1}{2} + \frac{1}{3} > \frac{1}{3} + \frac{1}{3}$

d) $\frac{1}{4} + \frac{1}{2} < \frac{1}{3} + \frac{1}{2}$

e) $\frac{2}{3} + \frac{1}{2} < \frac{3}{4} + \frac{1}{2}$

f) $\frac{2}{3} > \frac{1}{2} + \frac{1}{4}$

5. Look at the number line and inequalities below.



$x > y$ | $x < 1$ | $1 < x + y < 1\frac{1}{2}$

Write a possible value for ***x*** if:

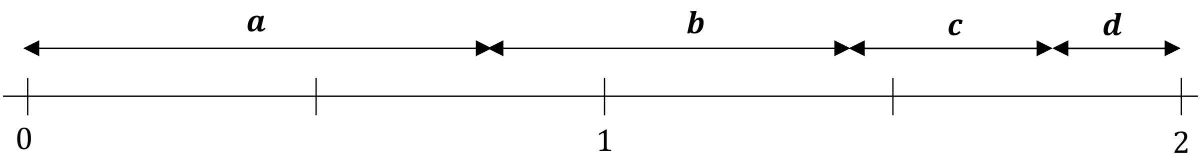
a) $y = \frac{1}{2}$

b) $y = \frac{2}{3}$

c) $y = \frac{1}{4}$

Questions for depth:

1. Look at the number line, inequalities and equation below.



$a < 1$ | $a > b > c > d$ | $a + b + c + d = 2$

a) If $\boldsymbol{a} = \frac{5}{6}$, write down a set of possible values for ***b***, ***c*** and ***d***.

b) If $\boldsymbol{b} = \frac{2}{3}$, write down a set of possible values for ***a***, ***c*** and ***d***.

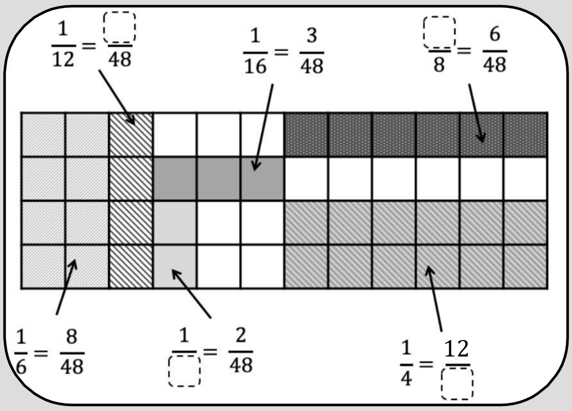
c) If $\boldsymbol{c} = \frac{1}{3}$, write down a set of possible values for ***a***, ***b*** and ***d***.

d) If $\boldsymbol{d} = \frac{1}{5}$, write down a set of possible values for ***a***, ***b*** and ***c***.

Session 3: Using common denominators

Concept Corner

We can accurately add or subtract any _______________ by using a common denominator.



4	fractions
48	written
24	1, 11

Using this diagram we can see that $\frac{1}{6}$ and $\frac{1}{16}$ can both be _________ in the form $\frac{x}{48}$.

$$\frac{1}{6} + \frac{1}{16} = \frac{8}{48} + \frac{3}{48} = \frac{\square}{48}$$

1. Copy and complete the sets of equivalent fractions.

a) $\frac{2}{3} = \frac{\ }{9} = \frac{\ }{12}$

b) $\frac{\ }{5} = \frac{\ }{20} = \frac{4}{10}$

c) $\frac{2}{7} = \frac{\ }{21} = \frac{4}{\ }$

d) $\frac{30}{\ } = \frac{10}{8} = 1\frac{\ }{4}$

e) $\frac{84}{\ } = \frac{7}{11} = \frac{\ }{33}$

f) $\frac{84}{\ } = \frac{7}{11} = \frac{\ }{33}$

2. Copy and complete the fraction calculations below

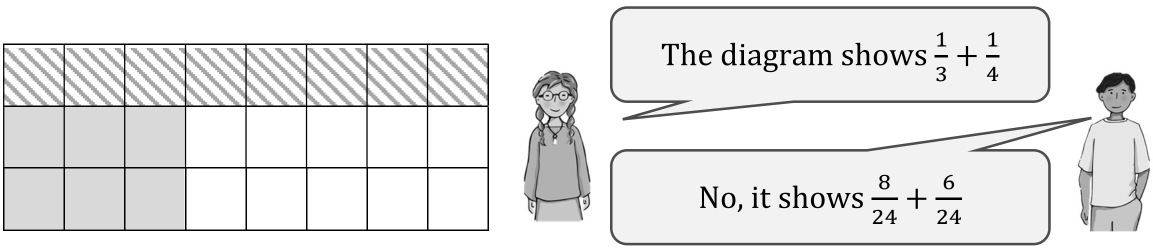
a) $\frac{1}{2} + \frac{1}{4} = \frac{\ }{4} + \frac{1}{4} =$

b) $\frac{1}{4} + \frac{1}{8} = \frac{\ }{8} + \frac{\ }{8} =$

c) $\frac{2}{3} - \frac{1}{6} = \frac{\ }{6} - \frac{1}{6} =$

d) $\frac{3}{4} + \frac{5}{12} = \frac{\ }{12} + \frac{5}{12} =$

3. In the diagram below, the whole rectangular grid represents 1.



Which student do you agree with? Explain your answer.

4. Look at the rectangular grid in question 3.

a) What is the answer to the fraction addition represented in question 3?

b) Draw your own 24-square rectangular grids representing 1, and shade them in to represent and work out the answer to these fraction calculations:

i) $\frac{1}{3} + \frac{5}{8} =$ ii) $\frac{5}{12} + \frac{3}{8} =$ iii) $\frac{7}{8} - \frac{5}{6} =$

5.

a) Find the lowest common multiples of the following pairs of numbers.

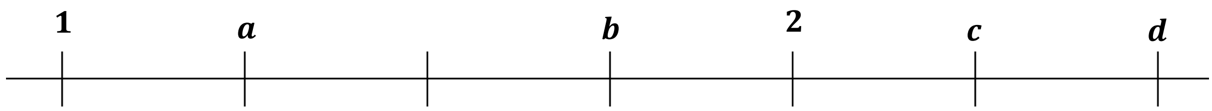
i) 8 and 10 ii) 12 and 3 iii) 7 and 4

b) Complete the fractions calculations below. Your answers to 5a) might help.

i) $\frac{5}{8} + \frac{3}{10} =$ ii) $\frac{11}{12} - \frac{2}{3} =$ iii) $\frac{2}{7} - \frac{3}{4} =$

c) Explain the connections between your answers to 5a) and your methods of calculation in 5b)

6. Look at the number line below. Complete each calculation in **two different** ways (not using equivalent fractions).



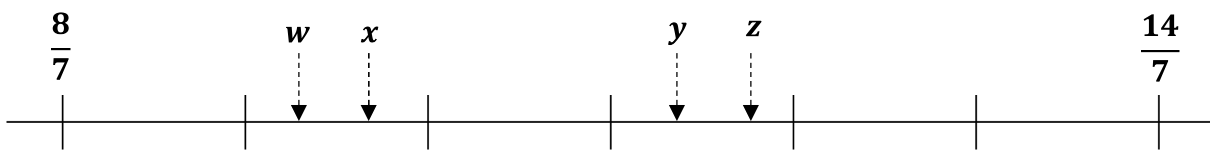
a) $\frac{5}{8} + \square =$ a number between a and b

b) $\frac{7}{5} + \square =$ a number between b and c

c) $\frac{20}{7} - \square =$ a number between c and d

Questions for depth:

1. Look at the number line below.



Write down **two different sets** of values for w-z such that $w + x + y + z = 6$

Session 4: Distributivity

Concept Corner

We can use our understanding of ___________ to help __________ fraction calculations.

For example:

$$\frac{3}{8} \times 7 + \frac{1}{8} \times 7 = \left(\frac{3}{8} + \_\_\right) \times 7 = \frac{1}{2} \times 7$$

$$3 \times \frac{2}{3} - 3 \times \frac{1}{6} = \left(\_\_ - \frac{1}{6}\right) \times 3 = \frac{1}{2} \times 3$$

simplify

distributivity

$\frac{1}{8}$ $\frac{2}{3}$

1. Copy can complete the following calculations.

a) $\frac{2}{3} - \frac{1}{6} =$

b) $\quad = \frac{3}{5} + \frac{1}{2}$

c) $\frac{5}{6} + \quad = \frac{11}{12}$

d) $\frac{5}{3} - \frac{11}{15} =$

e) $\frac{1}{6} = \quad - \frac{2}{9}$

f) $\frac{4}{9} + \frac{2}{5} =$

2. Copy and complete the calculations below:

a) $\frac{2}{3} \times 7 + \frac{1}{5} \times 7 = (\_\_ + \_\_) \times 7$

b) $\frac{2}{7} \times 3 - 3 \times \frac{1}{4} = 3 \times (\_\_ - \_\_)$

c) $8 \times \_\_ - \frac{1}{9} \times 8 = \_\_ \times \left(\frac{3}{5} - \frac{1}{9}\right)$

d) $\frac{7}{12} \times 10 - 10 \times \frac{1}{\square} = 5$

3. Decide if each of the statements below are true or false:

e) $\frac{2}{3} \times 3 + \frac{1}{4} \times 3 > 3$

f) $\frac{4}{3} \times 12 - 12 \times \frac{1}{4} > 12$

g) $8 \times \frac{2}{3} - \frac{1}{4} \times 8 > 4$

h) $\frac{3}{5} \times 10 - 10 \times \frac{1}{6} > 5$

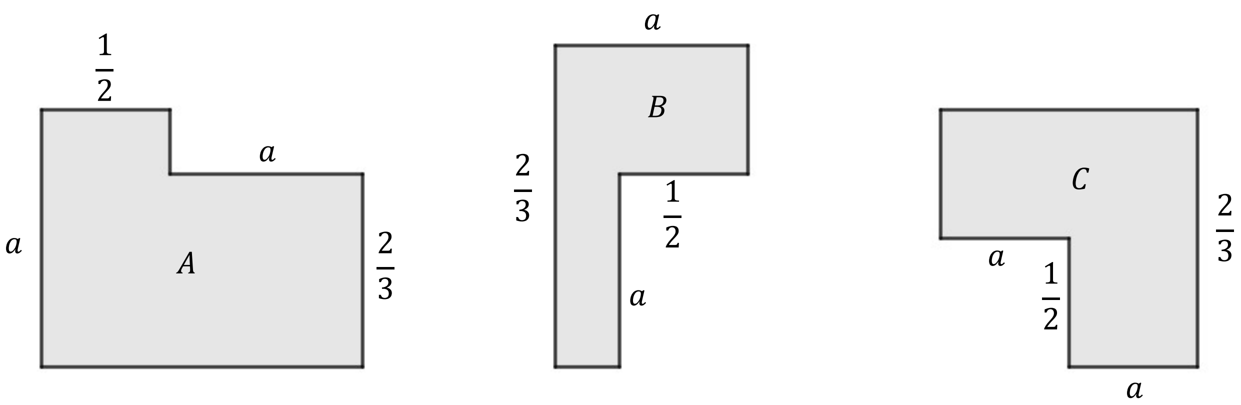
4. Match the cards showing equivalent calculations.

$\frac{1}{2} \times 5 - 5 \times \frac{3}{4}$	$5 \times \frac{1}{4}$	$\frac{3}{4} \times 5$	$\frac{5}{2} + \frac{1}{4}$
$5 \times \frac{1}{2} - \frac{1}{4} \times 5$	$\frac{1}{2} \times 5 + \frac{1}{4}$	$5 \times \left(\frac{1}{2} - \frac{3}{4}\right)$	$\frac{1}{2} \times 5 + \frac{1}{4} \times 5$

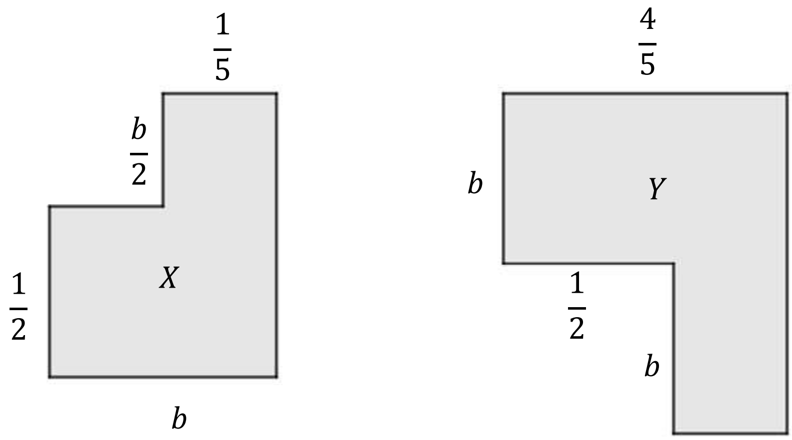
5. Work out the answer to each pair of cards from question 3.

6. The cards below show the area of each shape. Match then to the correct shape.

$\frac{1}{6} \times a$	$a \times \frac{5}{6}$	$a \times \frac{7}{6}$



7. Work out an expression to calculate the area of each shape below. Leave your answers in the form $\frac{\square}{\square} \times b$ or similar.



Questions for depth

1. Draw two different hexagons similar to those in questions 7 and 8 that both have an area of $\frac{7b}{8}$.

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Week 10: Percentage

Session 1: Percentage number line

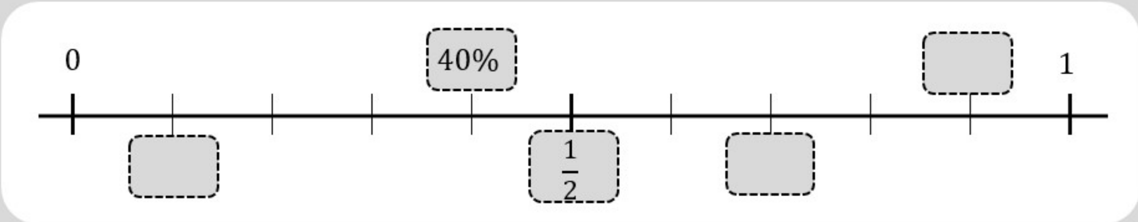
Concept Corner

We can represent fractions as ______________.

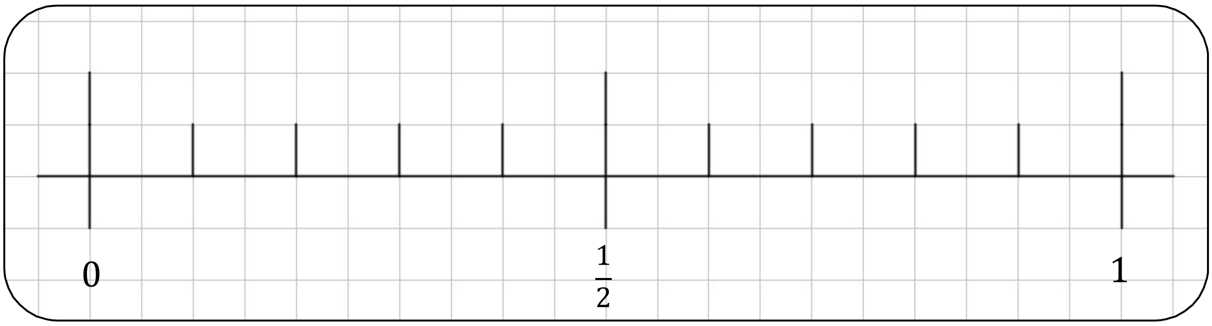
Percentage is another way of writing numbers as fractions with a ______________ of 100.

We can show these percentages on a ______________ line as in the example below.

70% | denominator | $\frac{9}{10}$ | number | 10% | percentages



1. Copy the number line below and place the numbers shown onto it.



90% | 60% | $\frac{3}{10}$ | $\frac{2}{5}$ | 25%

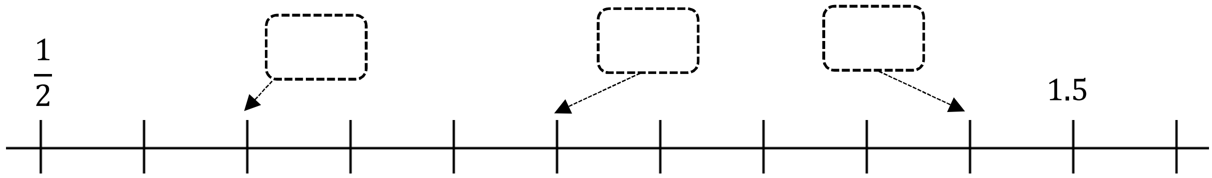
2. Write the numbers below in percentage form.

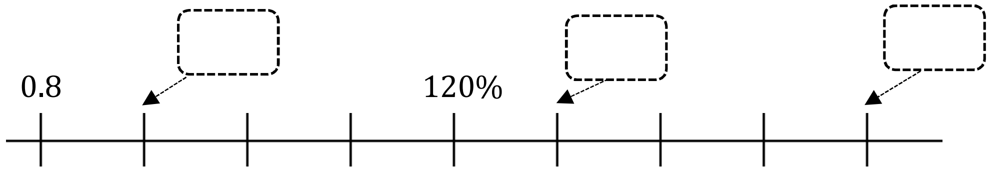
a) $\frac{1}{4}$ b) 0.5 c) 0.8 d) $\frac{3}{10}$

e) 0.15 f) $\frac{3}{4}$ g) $\frac{2}{5}$ h) $\frac{3}{2}$

3. Write down **two different numbers** that lie between $\frac{1}{2}$ and 40%.
4. Write your answers to question 3 in a different form (e.g. fraction, percentage, decimal).

5. Write down the numbers missing from the boxes on the number lines in **fraction, percentage and decimal** form.

a) 

b) 

6. Write down the number that is **halfway between**:

a) $\frac{1}{4}$ and 75%

b) 0.8 and 100%

c) $\frac{4}{5}$ and 120%

d) 0.5 and $\frac{9}{10}$

e) 140% and $\frac{7}{10}$

f) 0.1 and 100%

7. Place the numbers below into the two groups shown.

0.5 | 0.054 | $\frac{4}{10}$ | $\frac{45}{10}$ | $\frac{100}{45}$ | 4.5%

Numbers < 45%

Numbers > 45%

Questions for depth:

1. Decide if the fractions below are **greater than** or **less than** 55%. Explain how you know.

$\frac{11}{21}$ | $\frac{12}{20}$ | $\frac{11}{201}$ | $\frac{12}{21}$

2. Place the four numbers from question 1 above in **ascending** order.

3. Look at your list of numbers from question 2 above. Write numbers in **percentage form** that lie between the numbers in ascending order.

Session 2: Tenths, hundredths and thousandths

Concept Corner

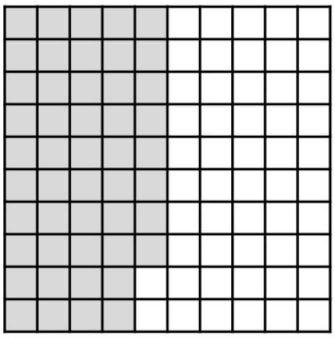
Tenths, hundredths and ______________ can be written in different ways, including as _____________.

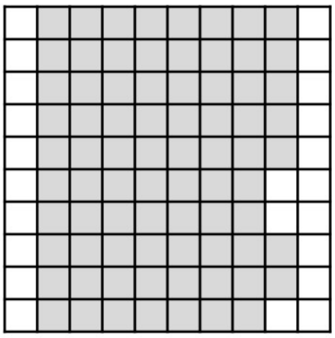
Fractions	Decimal	Percent
$\frac{\square}{10} + \frac{3}{100} + \frac{\square}{1000} = \frac{235}{1000}$	0.235	$\square$

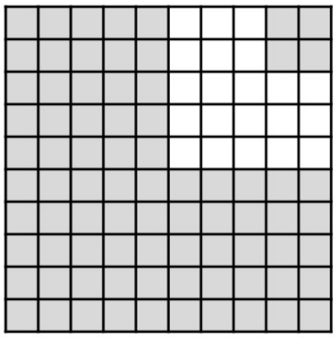
We can also represent these numbers using a number line.

Word bank:
- percentages
- thousandths
- 23.5%
- 3
- 2
- 0.235

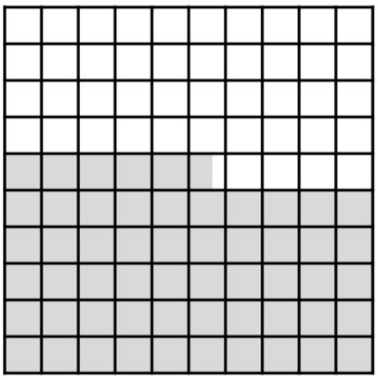
1. The 100-squares below each represent 1.
   What numbers are represented by the shaded sections. Write your answers as **fractions, decimals** and **percentage**.

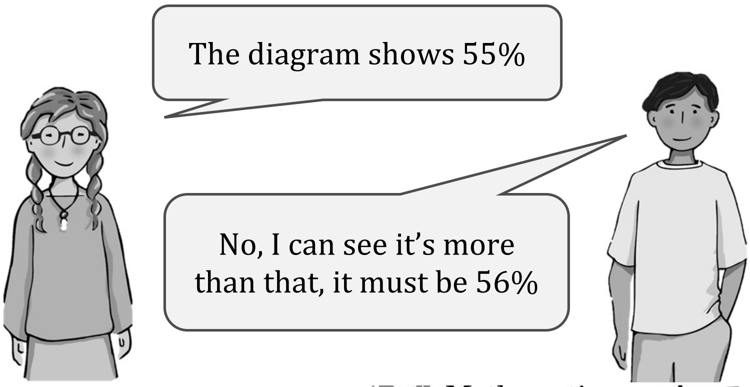
a) 

b) 

c) 

2. The 100-square below represents 1.
   Look at the two students' statement. Do you agree with either of the students? Explain why or why not.

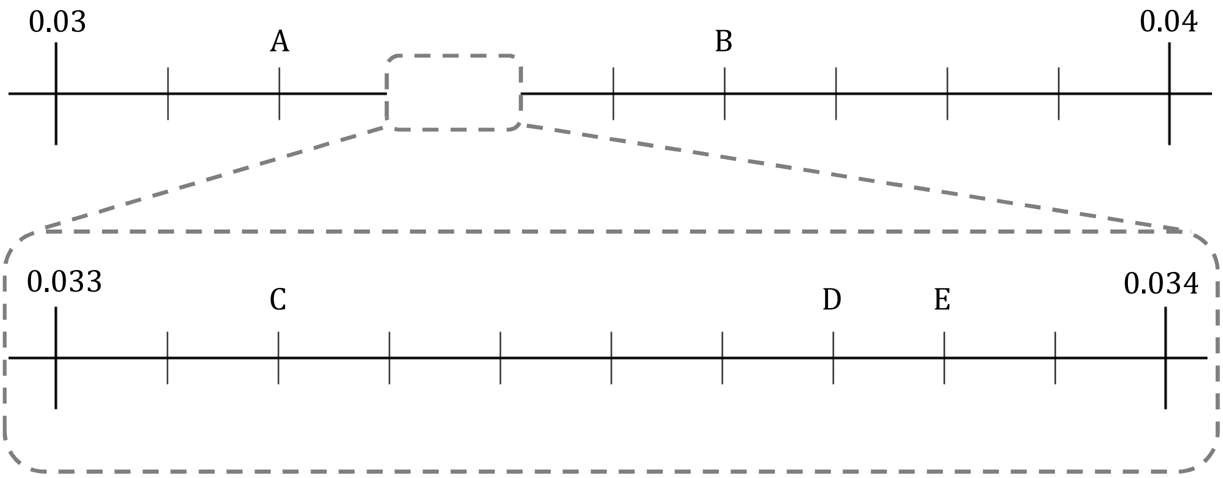




3. Look at the table below. The first row has been completed as an example.
   Copy and complete the remainder of the table.

Fraction	Different equivalent fraction	Sum of tenths, hundredths and thousandths	Decimal	Percentage
$\frac{3}{5}$	$\frac{60}{100}$	$\frac{6}{10}+\frac{0}{100}+\frac{0}{1000}$	0.6	60%
		$\frac{1}{10}+\frac{0}{100}+\frac{5}{1000}$		
$\frac{13}{20}$				
				12.5%
	$\frac{918}{1000}$			
		$\frac{0}{10}+\frac{3}{100}+\frac{7}{1000}$		

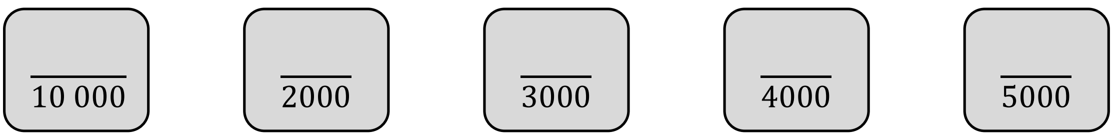
4. The images below show a 'zoomed in' view of part of a number line. Write down the numbers that lie at points A-D as shown.



5. Write your answers to question 4. as percentages.

Questions for depth:

1. Complete the number cards below to form numbers that lie between D and E in question 4 above.



2. Put your answers to question 1 in ascending order.

Session 3: Converting fractions and percentage

Concept Corner

We can convert between fractions and percentages using our understanding of ____________ fractions.

60	equivalent			
20	100	3		
decimal place				

$$\frac{3}{5} = \frac{\square}{100} = 60\%$$

$$45\% = \frac{45}{\square} = \frac{9}{\square}$$

Some fractions have recurring or repeating ____________ digits when written as a percentage or decimal. E.g.:

$$\frac{1}{\square} = 0.33\dot{3} = 33.\dot{3}\%$$

1. Write the fractions below as percentages.

a) $\frac{3}{4}$ b) $\frac{1}{5}$ c) $\frac{7}{20}$ d) $\frac{23}{50}$

e) $\frac{7}{10}$ f) $\frac{107}{100}$ g) $\frac{24}{25}$ h) $\frac{12}{40}$

i) $\frac{2}{3}$ j) $\frac{11}{10}$ k) $\frac{545}{1000}$ l) $\frac{7}{5}$

2. Write the percentages below as fractions in their simplest form.

a) 30% b) 80% c) 85% d) 45%

e) 12% f) $66.\dot{6}\%$ g) 125% h) 120%

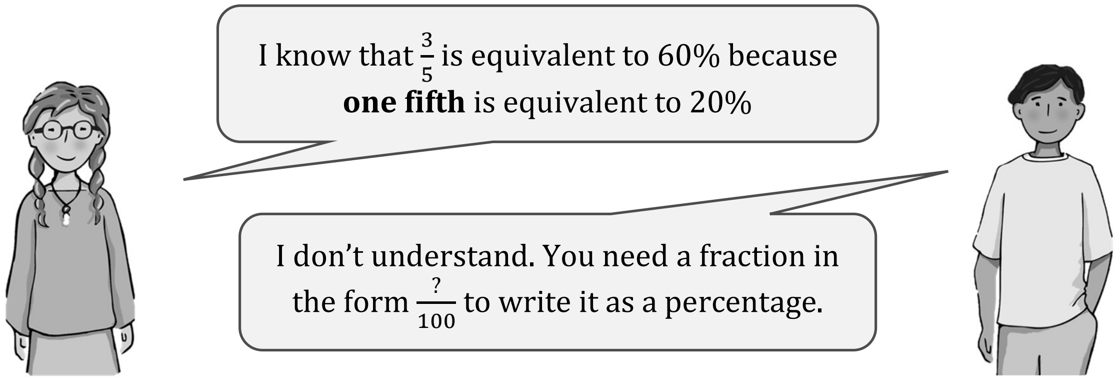
3. Match each fraction card with a percentage card. Complete the cards where necessary.

$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{3}$
$\frac{1}{\ }$	$\frac{1}{4}$	$\frac{2}{\ }$

__%	37.5%	$66.\dot{6}\%$
20%	12.5%	25%

4. Put the fraction cards from question 3 in **descending** order.

5. One student is trying to explain their approach for writing $\frac{3}{5}$ in percentage form. Draw a diagram (e.g. a bar model) to help explain.



6. Decide if the statements below are true or false.

a) $\frac{9}{4} = 225\%$

b) $\frac{15}{4} = 400\%$

c) $\frac{5}{3} > 200\%$

d) $\frac{10}{3} > 300\%$

e) $\frac{7}{5} > 200\%$

f) $\frac{9}{5} > 200\%$

7. Decide if the numbers in each box are the same or different. Explain how you know.

a) $\frac{4}{5}$, 0.8, 80%

b) $\frac{3}{8}$, $3 \times 12.5\%$, 1.25

c) 400%, 4×100, $\frac{28}{7}$

d) $\frac{9}{4}$, 0.25×9, 2.25%

Questions for depth:

1. How many solutions to question 8 are there if you can only use each digit card once?

2. How many solutions to question 8 are there if you only have the number cards 0, 1, 2, 3, 5, 6 and can use them only once each?

Session 4: Percentages of quantities I

Concept Corner

We can use our understanding of ____________ to help calculate with percentages.

The example below shows a set of ______________ calculations and statements.

Word bank: equivalent | 0.75 | 100 | 3 | fractions

75% of £150 $\qquad$ ☐ × £150 $\qquad$ $\frac{75}{☐}$ × £150 $\qquad$ $\frac{3}{4}$ × £150

1. Write the percentages below as fractions in their simplest form:

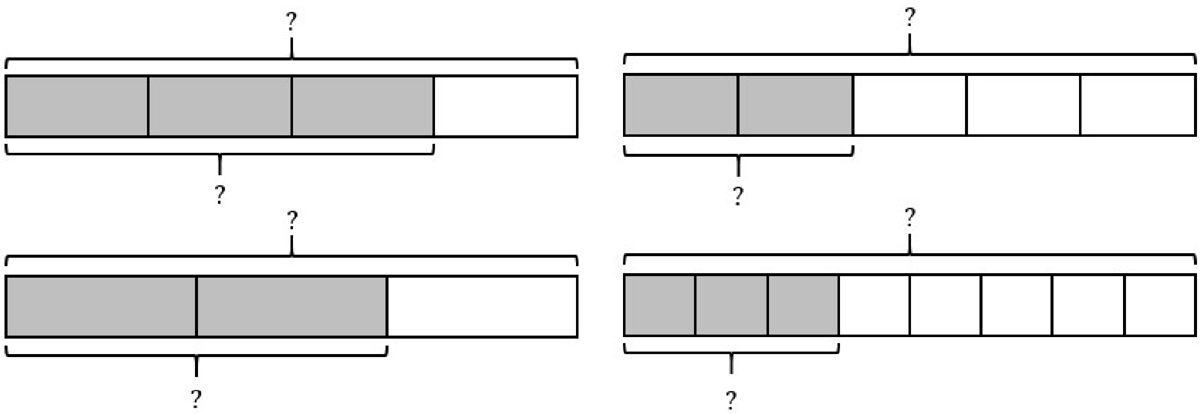
a) 50% $\qquad$ b) 10% $\qquad$ c) 40%

d) 65% $\qquad$ e) 15% $\qquad$ f) 12.5%

2. Calculate:

a) 30% of £50 $\qquad$ b) 60% of 150 cm $\qquad$ c) 85% of 250 kg

3. Match each bar model to a calculation. Complete the bar models and calculations.



40% of 120 = ___ $\qquad$ 66.$\dot{6}$% of 72 = ___ $\qquad$ 37.5% of ___ = 63 $\qquad$ 75% of 300 = ___

4. Re-write the calculation cards from question 2 as fraction calculations. E.g.:

75% of 300 = $\frac{3}{4}$ × 300

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5. Look at Phil's statement below. Explain why he is **wrong**. You can draw a diagram to help.

> To find 10% of a quantity I divide by 10, so:
>
> - To find 5% I must divide by 5;
> - To find 25% I must divide by 25.

6. Find the odd one out in each set below. Explain how you know it doesn't fit the set.

a) | 25% of 60 | 16% of 100 | 25% of 64 |
|---|---|---|

b) | $40\% \times 60$ | 20% of 120 | $80\% \times 40$ |
|---|---|---|

c) | $62.5\% \times n$ | $\frac{3}{8}$ of n | $\frac{625}{100} \times n$ |
|---|---|---|

7. Match up the **pairs** of equivalent calculations from the cards below.

50% of 196	100×0.99	0.75×132
$98 \times 50\%$	$147 \times \frac{1}{3}$	$200\% \times 49$

8. Complete the equations below

a) $20\% \times 15\% =$ ____% of 0.05

b) $\frac{1}{3} \times 15\% =$ ____% of $\frac{1}{6}$

c) $87.5\% \times \frac{8}{7} = \frac{4}{3} \times$ ____%

d) $0.6 \times$ ____ $= \frac{1}{5}$ of ____%

Questions for depth:

1. Write the following expressions in the form ***a*% of *n*.**

$\left(\frac{1}{2}\right)^2 \times 0.5 \times 3n$	$\left(\frac{2}{5}\right)^2 \times \left(\frac{1}{2}\right)^3 \times n$	$\left(\frac{3}{2}\right)^3 \times \left(\frac{1}{3}\right) \times n - \frac{n}{8}$