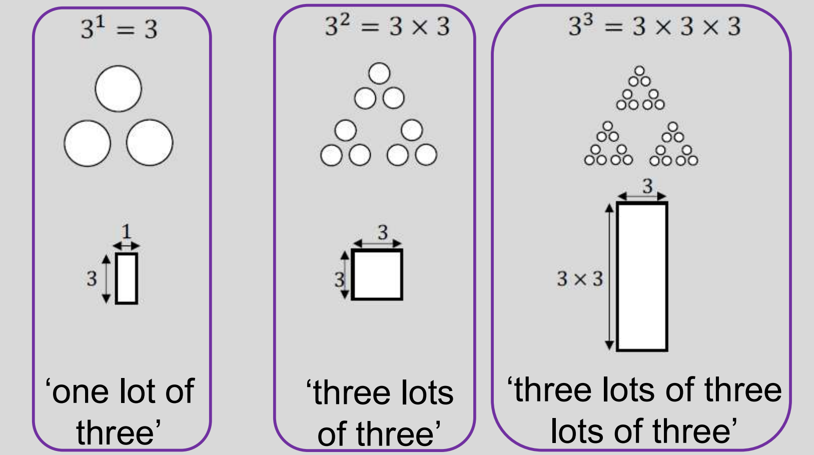
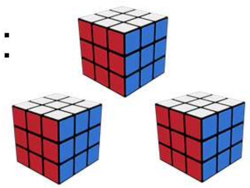
Answers: Week 3 Session 1

Task 1



3^4: E.g.:



'three cubes made up of 3 × 3 × 3 blocks'

Task 2

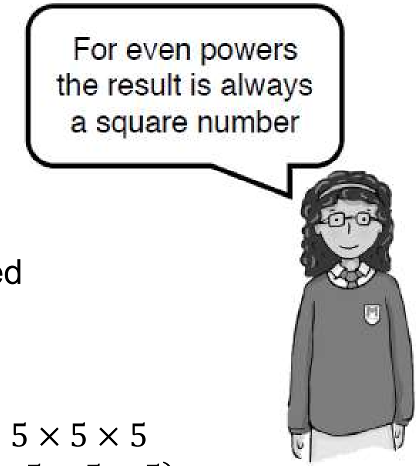
She is correct:
A square number is a number that is the product of a number multiplied by itself, e.g:

25 *is square* because it is the product of 5×5.

If a number has an **even** power, it can **always** be arranged into two equal factors that are multiplied together, e.g.:

$$\begin{aligned} &3^6 \\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= (3 \times 3 \times 3) \times (3 \times 3 \times 3) \\ &= 3^3 \times 3^3 \\ &= 27 \times 27 \end{aligned}$$

$$\begin{aligned} &5^8 \\ &= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ &= (5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) \\ &= 5^4 \times 5^4 \\ &= 625 \times 625 \end{aligned}$$



In both the examples, we know the product **must** be square.

Exercise

1.

a) 6^2

b) 6^4

c) 2^5

2.

6^6

3.

$9^1 = 9$

$5^3 = 25 \times 5$

$\cancel{5^3 = 3 \times 3 \times 3 \times 3 \times 3}$

$\cancel{10^2 = 2^{10}}$

$\cancel{8^3 = 24}$

$8^4 = 8 \times 8 \times 8 \times 8$

4.

a) $2^3 < 3^2$

b) $2^4 = 4^2$

c) $3^3 < 5^2$

d) $1^8 = 1^5$

5.

2 to the power of 4:	3 to the power of 3:	4 to the power of 2:	5 to the power of 2:
A C L	D J K	B E G	F H I

6.

$2 \times 2 \times 7 \times 7 \times 7$	$5^3 \times 3$
$5 \times 3 \times 5 \times 3$	$2^2 \times 3^2 \times 5^2 \times 7$
$7 \times 7 \times 5 \times 5 \times 3 \times 3 \times 5 \times 7$	$3^2 \times 5^2$
$5 \times 5 \times 3 \times 3 \times 2 \times 2 \times 7$	$2^2 \times 7^3$
$5 \times 5 \times 5 \times 3$	$3^2 \times 5^3 \times 7^3$

7.

$a = 5$

8.

E.g. 2, 4, 6

Any even numbers

D1.

a) 2^3

b) 3×5^2

c) $2^2 \times 3^2$

Answers: Week 3 Session 2

Task 1

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$3 = 3 \times 1$
$6 = 2 \times 3$
$9 = 3 \times 3$
$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
$24 = 2 \times 2 \times 2 \times 3$
$27 = 3 \times 3 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
$54 = 2 \times 3 \times 3 \times 3$
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$81 = 3 \times 3 \times 3 \times 3$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Task 2

Introducing 4s **will not change** the numbers that can be shaded because we have already shaded all the products of 2×2 that are on the grid.

Including 5s results in all multiples of 5 being shaded **except** those with a prime factor other than 2, 3 or 5.

E.g. numbers **not** shaded:

$35 = 7 \times 5$
$55 = 11 \times 5$
$60 = 13 \times 5$
$70 = 2 \times 5 \times 7$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Exercise

1.

Blue shading indicates various possible answers

a) $24 = 3 \times 8$
$24 = 3 \times 2 \times 4$
$24 = 3 \times 2 \times 2 \times 2$

b) $120 = 10 \times 12$
$120 = 5 \times 2 \times 12$
$120 = 5 \times 2 \times 2 \times 6$

2.

a) $12 = 2 \times 3 \times 2$
b) $20 = 2 \times 2 \times 5$
c) $30 = 2 \times 3 \times 5$
d) $36 = 2 \times 2 \times 3 \times 3$
e) $45 = 3 \times 3 \times 5$
f) $54 = 2 \times 3 \times 3 \times 3$

3.

a)
$24 = 2 \times 2 \times 2 \times 3$

b)
$48 = 2 \times 2 \times 2 \times 2 \times 3$

c)
$60 = 2 \times 2 \times 3 \times 5$

d)
$72 = 2 \times 2 \times 2 \times 3 \times 3$

4.

$2 \times 2 \times 3 = 12$

$2 \times 2 \times 5 = 20$

$2 \times 3 \times 3 = 18$

$2 \times 3 \times 5 = 30$

$3 \times 3 \times 5 = 45$

5.

a)
i) $28 = 2 \times 2 \times 7$ or 4×7
ii) $63 = 3 \times 3 \times 7$ or 9×7
iii) $42 = 2 \times 3 \times 7$ or 6×7

b) No, she cannot form multiples of 11 that are also multiples of **prime numbers** missing from Gavin's list, e.g. she cannot form 11×13.

c) He can remove factors that are **not prime** as they can be formed by the prime factors in the list:
$4 = 2 \times 2$ $6 = 2 \times 3$ $8 = 2 \times 2 \times 2$
$9 = 3 \times 3$ (see part a) for examples)

D1.

1, 2, 5, 7, 13, 41, 97

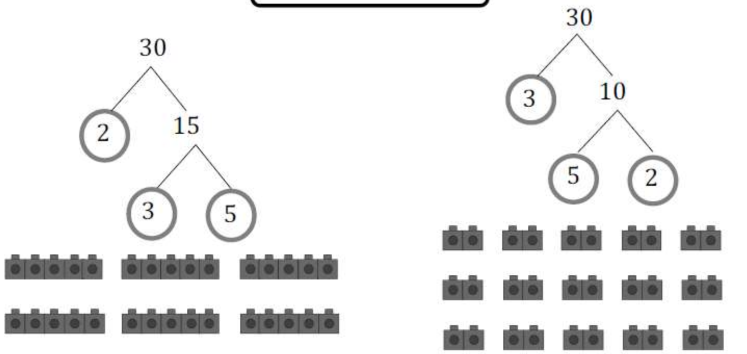
D2.

104, 105, 112

Answers: Week 3 Session 3

Task 1

$30 = 2 \times 3 \times 5$

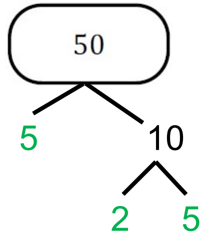


Same e.g: Prime factors, number of cubes
Different e.g.: Initial factor pair chosen, order of prime factors in the tree

Whatever factor pair we start with we will always end up with the same prime factors. **Try this out for yourself with different starting numbers.**

Task 2

We can use the **product of primes** for some numbers to help us work them out for others. We just need to look at the connection between the numbers: E.g.:

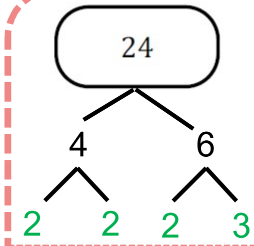


So, $50 = 2 \times 5 \times 5$

150: $150 = 50 \times 3$
So, $150 = 2 \times 5 \times 5 \times 3$

50×10: $50 \times 10 = 50 \times 2 \times 5$
So, $50 \times 10 = 2 \times 5 \times 5 \times 2 \times 5$

50^2: $50^2 = 50 \times 50$
So, $50 \times 10 = 2 \times 5 \times 5 \times 2 \times 5 \times 5$



So, $24 = 2 \times 2 \times 2 \times 3$

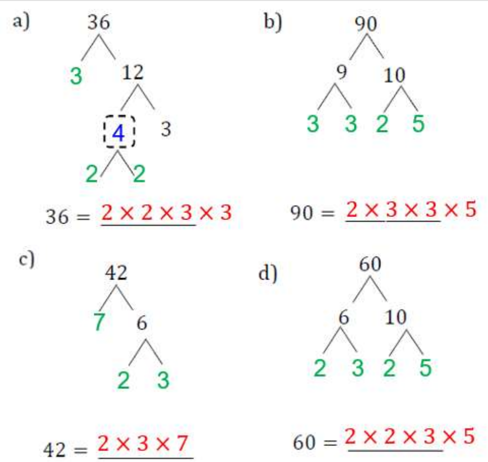
72: $72 = 24 \times 3$
$72 = 2 \times 2 \times 2 \times 3 \times 3$

24×10: $24 \times 10 = 24 \times 2 \times 5$
$24 \times 10 = 2 \times 2 \times 2 \times 3 \times 2 \times 5$

24^2: $24^2 = 24 \times 24$
$24^2 = 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 3$

Exercise

1.



a) $36 = 2 \times 2 \times 3 \times 3$

b) $90 = 2 \times 3 \times 3 \times 5$

c) $42 = 2 \times 3 \times 7$

d) $60 = 2 \times 2 \times 3 \times 5$

2.

a) 72 $= 2 \times 2 \times 2 \times 3 \times 3$

b) 175 $= 5 \times 5 \times 7$

c) 144 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3$

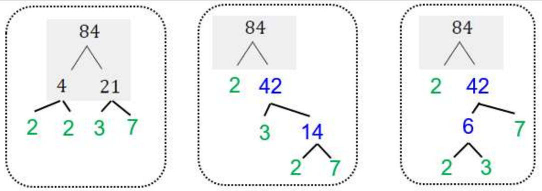
d) 1750 $= 2 \times 5 \times 5 \times 5 \times 7$

e) 350 $= 2 \times 5 \times 5 \times 7$

f) 216 $= 2 \times 2 \times 2 \times 3 \times 3 \times 3$

3.

Brenda is correct – it is possible to start with any factor pair, the final product of primes will be the same:



$84 = 2 \times 2 \times 3 \times 7$

4.

a) All share the prime factors of 72;
then $\times\ 2$ for 144 in c);
or $\times\ 3$ for 216 in f).

b) All share the prime factors for 175;
then $\times\ 2$ for 350 in e);
and $\times\ 2 \times 5$ for 1750 in f)

c) $72 \times 10 = 720$ so we must multiply the prime factors of 72 by $\times\ 2 \times 5$;
720
$= 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 5$

5.

These can be answered by comparing the index of 2, 3 and 5 in each multiplication. There is **no need** to calculate the products.

a) True – 2^2 for $\boldsymbol{d}$ versus 2 for $\boldsymbol{a}$
b) False – $\boldsymbol{c}$ is 2^2 times the value of $\boldsymbol{b}$
c) True – $2^3$and 5^3 for $\boldsymbol{c}$ versus 2^2 and 5^2 for $\boldsymbol{d}$
d) False – Comparing both to $2 \times 3^2 \times 5^2$ shows $\boldsymbol{b}$ has greater index for 5 versus $\boldsymbol{d}$ has greater index for 2.

D1.

a) False
b) False
c) True

Answers: Week 3 Session 4

Task 1

Shows prime factors (green highlighting)

$1 = 1$
$2 = 2$
$3 = 3$
$4 = 2 \times 2$
$5 = 5$
$6 = 2 \times 3$
$10 = 2 \times 5$
$12 = 2 \times 2 \times 3$
$15 = 3 \times 5$
$20 = 2 \times 2 \times 5$
$30 = 2 \times 3 \times 5$
$\mathbf{60 = 2 \times 2 \times 3 \times 5}$

Comparing the prime factorisation of 60 to the prime factorisations of its factors:

- Prime factorisation of any factor is a **subset** or **part of** the prime factorisation of 60.
- Prime factorisation of factor pairs (E.g. 6 and 10) forms the prime factorisation of 60 when multiplied together.
- All prime factorisations are composed of the prime factors.

Task 2

We can identify **factor pairs** by grouping the prime factorisation into **two groups** in different ways.

For example, if our number's prime factorisation is:

$$2 \times 5^2 \times 7$$

we know that multiplying these numbers together gives our number (350).

We also know that we can group the prime factors differently:

$$(2 \times 5) \times (5 \times 7) = 10 \times 35$$

So 10 and 35 must be a factor pair of our number.

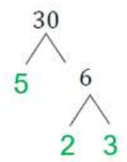
$2 \times 5^2 \times 7$

Factor pairs:

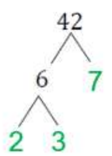
$2 \times 5 \times 7$	5	70 and 5
5×7	5×2	35 and 10
$5^2 \times 7$	2	175 and 2
2×5^2	7	50 and 7
2×7	5^2	14 and 25
$2 \times 5^2 \times 7$	1	350 and 1

Exercise

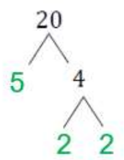
1.

a)


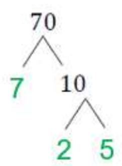
$30 = 2 \times 3 \times 5$

b)


$42 = 2 \times 3 \times 7$

c)


$20 = 2 \times 2 \times 5$

d)


$70 = 2 \times 5 \times 7$

2.

a) 30: 1, 2, 3, 5, 6, 10, 15, 30 – 8 factors
42: 1, 2, 3, 6, 7, 14, 21, 42 – 8 factors
20: 1, 2, 4, 5, 10, 20 – 6 factors
70: 1, 2, 5, 7, 10, 14, 35, 70 – 8 factors

b) **30, 42 and 70 all have 8 factors:**
- 2 factors: The original number and 1
- 3 factors: The distinct prime factors
- 3 factors: 3 ways of combining the prime factors to form new factors (e.g. for 42 forming $3 \times 7 = 21$)

20 only has 6 factors:
- 2 factors: The original number and 1
- 2 factors: The distinct prime factors (2 and 7)
- 2 factors: 2 ways of combining the prime factors to form new factors ($2 \times 2 = 4$ and $2 \times 7 = 14$)

c) Any number with prime factors in the form $a \times a \times b$:
E.g. 45 has 6 factors because we can write it $3 \times 3 \times 5$

3.

a) 5
b) Yes: $2 \times 2 = \mathbf{4}$
c) $168 \times 42 = 2^4 \times 3^2 \times 7^2 = (2^2 \times 3 \times 7) \times (2^2 \times 3 \times 7)$ so it must be a square number
d) $2^8 \times 3^6 \times 5^4 \times 7^4$

4.

She is correct: Any factor pair of 270 is the product of 2, 3, 3, 3 and 5 split into two sets. Any set involving 2 will be even. Any set without 2 will be odd. Therefore one even and odd factor in each pair (as well as 1×270).

5.

210 has most (16 factors)

230 has fewest (8 factors)

6.

220 has the greatest factor:

$2 \times 5 \times 11$
$= \mathbf{110}$

D1.

a)
Greatest (omit 3):
$a = 2^5 \times 3^2 \times 7^2$
$= 14\,112$

Least (omit odds):
$a = 2^5 = 32$

b)
Greatest (omit 2^3):
$b = 2^2 \times 3^3 \times 7^2$
$= 5292$

Least (omit $2^3 \times 3^3$):
$b = 2^2 \times 7^2 = 196$