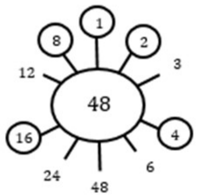
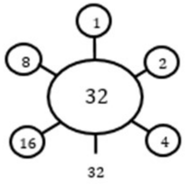
Answers: Week 4 Session 1

Task 1



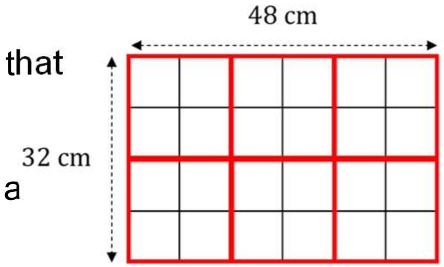


Common factors are numbers that are factors of both numbers. The common factors of 32 and 48 are circled above. The **highest common factor (HCF)** is simply the greatest number, which in this case is 16.

1 is the lowest common factor of **every** pair of numbers, so the lowest common factor is not an interesting or important thing to find.

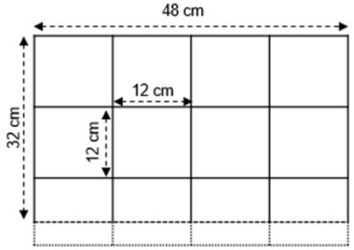
It's worth noticing that all the other common factors are factors of 16, will this be the case when looking at other HCFs?

Task 2



The squares have sides of 8 cm. the largest square that fits has sides of 16 cm (see diagram).

Other squares that will fit perfectly are squares with a side length of:
1 cm, 2 cm, 4 cm.



Notice the similarities between this task and Task 1. The squares that fit are the common factors of 48 and 16.

Other squares will not fit both horizontally **and** vertically.

Squares that fit in a 12 cm × 24 cm rectangle: 1 cm, 2 cm, 4 cm, 6 cm, **12 cm**
Squares that fit in a 18 cm × 24 cm rectangle: 1 cm, 2 cm, 3 cm, **6 cm**

Exercise

1.

a) 1, 2, 4, 7, 14, 28

b) 1, 2, 3, 6, 9, 18, 27, 54

c) 1, 2, 3, 4, 6, 9, 12, 18, 36

2.

a) 1, **2**

b) 1, 2, **4**

c) 1

d) 1, 2, 3, 6, 9, **18**

3.

Boldened in the Q2 answers.

a) 6 cm
b) 1 cm, 2 cm, 3 cm, 6 cm, 9 cm, 18 cm (note the similarity between this answer and the answer for 2d)

4.

e.g. 12 and 24, 24 and 36, 24 and 60, 36 and 84.

Both numbers **must** be a multiple of 12. But watch out for numbers with a higher common factor! For example, 24 and 48 have HCF = 24

5.

a) to d) = 6
e) to h) = 8

6.

180 and 252 much be shared into the same numbers of boxes. This is only possible if the number of boxes is a common factors of 180 and 252.

180: **1**, **2**, **3**, **4**, 5, **6**, **9**, 10, **12**, 15 **18**, 20, 30, **36**, 45, 60, 90, 180
252: **1**, **2**, **3**, **4**, **6**, 7, **9**, **12**, 14, **18**, 21, 28, **36**, 42, 63, 84, 126, 252

The number of boxes could be any of the bold, underlined numbers.

7.

a) Any two numbers that are a multiple of 16:
16, 32, 48, 64, 80, 96…. Etc.

b) In many cases the HCF will be 16, however in some cases the HCF will be a multiple of 16

e.g. HCF(64, 96) = 32, HCF(48, 96) = 48, HCF (128, 196) - 64

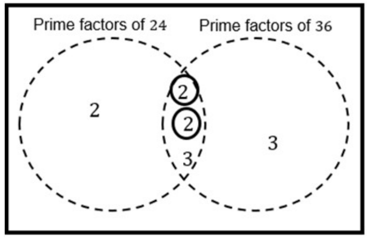
D1.

Any cube with a side length which is a common factor of 8, 12 and 16:
1, 2, 4

Answers: Week 4 Session 2

Task 1

We can find the common factors of 24 and 36 by looking at the common **prime** factors of 24 and 36.



Both 24 and 36 are ‘built up’ of two 2s and a 3.

We can find the common factors by making combinations* of these numbers

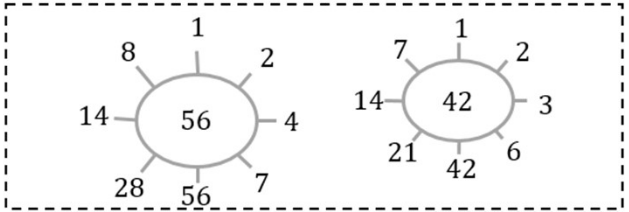
*Note that prime factors multiply to make a number, so when we combine them we multiply them.

2 = **2**
3 = **3**
2 × 2 = **4**
2 × 3 = **6**
2 × 2 × 3 = **12**

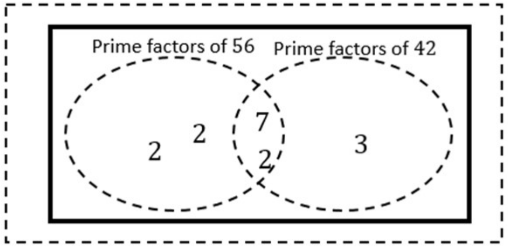
Interesting! The HCF is 12, and was found by multiplying all of the prime factors…

Task 2

This diagram lists all the factors, it shows a bit more than the other two diagrams but could be more time consuming. Imagine using this diagram to list all the factors of 900 and 1458!



Both of these diagrams show the prime factorisations of 56 and 42. The Venn diagram emphasises the common prime factors of 56 and 42.



$56 = 2 \times 2 \times 2 \times 7$

$42 = 2 \times 3 \times 7$

Exercise

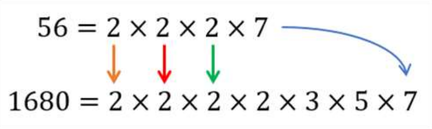
1.

a) 130 = 2 × 5 × 13
b) 104 = 2 × 2 × 2 × 13
c) 56 = 2 × 2 × 2 × 7
d) 308 = 2 × 2 × 7 × 11

2a.

a) i) Yes
ii) Yes
iii) Yes (2 × 7)
iv) No
v) Yes (2 × 2 × 2)
vi) No
vii) No
viii) Yes (2 × 2 × 2 × 2 × 3)

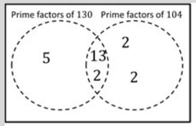
2b.

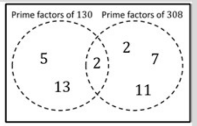


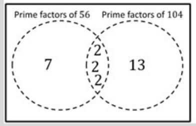
$56 = 2 \times 2 \times 2 \times 7$

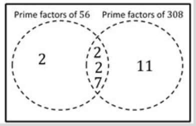
$1680 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7$

3.









4.

a) 26
b) 2
c) 8
d) 28
e) 2
f) 4

5.

Number 1	Number 2	HCF	Number 1	Number 2	HCF
140	12	4	60	70	10
84	20	4	210	28	14
60	28	4	140	42	14
210	12	6	84	70	14
84	30	6	60	105	15
60	42	6	84	105	21
210	20	10	140	105	35
140	30	10			

6.

After two multiples of 72 greater than 1000 which don't share another common factor (other than one).
e.g. 1782 and 1800
1782 = 24 x 72
1800 = 25 x 72

24 and 25 do not share any prime factors

7.

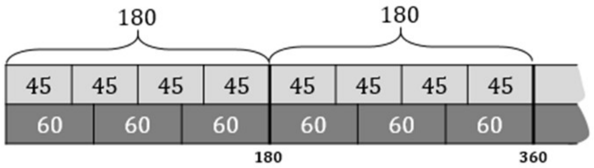
Squares with a side length of 21 m

D1.

The HCF of a and b can be expressed as a factor of $\frac{a \times b}{n}$, the HCF a and a + b can be expressed as $\frac{a \times (a+b)}{n}$. n will be the same value in both cases.

Answers: Week 4 Session 3

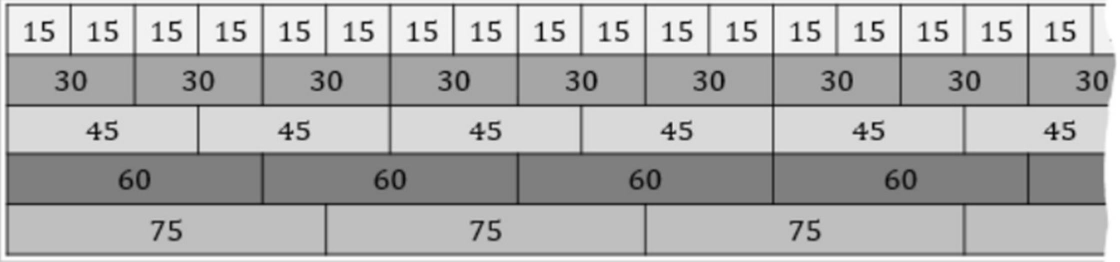
Task 1



The bars line up at 4 lots of 45, and 3 lots of 60. Therefore the lowest common multiple is 180. If the pattern is extended, we can see that bars line up **every** 180.

Task 2

LCM (15, 30) = 30
LCM (15, 45) = 45
LCM (15, 60) = 60
LCM (15, 75) = 75
LCM (30, 45) = 90
LCM (30, 60) = 60
LCM (30, 75) = 150
LCM (45, 60) = 180
LCM (45, 75) = 225
LCM (60, 75) = 300



Some thing to notice:
All of the numbers are multiples of 15 **and** all of the LCMs are multiples of 15
If one number is a multiple of another, the LCM will be the larger number (e.g. LCM(15,45) = 45, LCM(30,60) = 60

Greater depth
60 = **4** lots of 15, 75 = **5** lots of 15. The lowest common multiples of 4 and 5 is 20. The lowest common multiple of 60 and 75 is 300 or **20** lots of 15

Exercise

1.

a) 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144
b) 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108
c) 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84
d) 21, 42, 63, 84, 105, 126, 147, 168, 189, 210, 231, 252

2.

a) 36
b) 84
c) 84
d) 21
e) 63
f) 63

3.

a) e.g. 3 and 5, 3 and 15, 5 and 15
b) e.g. 3 and 7, 1 and 21, 7 and 21
c) e.g. 5 and 6, 2 and 15, 10 and 15
d) e.g. 4 and 9, 12 and 18, 12 and 18

Note some non-examples for 36. The lowest common multiple of 3 and 12 is **not** 36, even though the product of 3 and 12 is 36

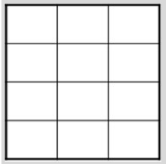
4.

68 years (the LCM of 4 and 17)

e.g. if they both hide in 2020…
…cicadas will appear in: 2037, 2054, 2071, 2088
…predators will appear in: 2024, 2028, 2032, 2036, 2040, 2044, 2048, 2052, 2056, 2060, 2068, 2072, 2076, 2080, 2084, 2088

5.

The smallest square will have sides of 180 cm. It is also possible to create squares with a side length that is a multiple of 180 (360, 540, 720 etc,)



6.

The smallest square will have a side length equal to the LCM of the two sides:
LCM(9, 24) = 72, LCM(9, 12) = 36, LCM(9, 36) = 36, LCM (24, 12) = 24, LCM (24, 36) = 36, LCM(12, 36) = 36

7.

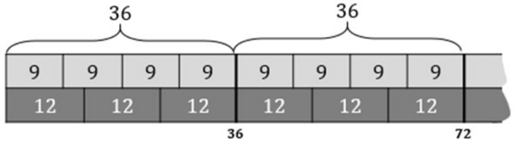
a) 84 seconds, A will have completed 7 full turns and B will have completed 4 full turns. 84 is the lowest common multiple of 12 and 21.
b) This will not change the answer

D1.

1) 28 seconds
2) No, they will cross every 28 seconds, and the crossings will be a third of the way around the turn. In terms of a clock, they will cross at 2 o'clock, 6 o'clock and 10 o'clock

Answers: Week 4 Session 4

Task 1



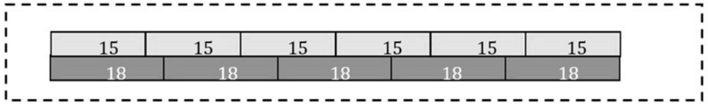
The bars line up at 36, and will continue to line up every 36. So the first five common multiples of 9 and 12 are:
36, 72, 108, 144, 180

9 = 3 × 3 **12** = 2 × 2 × 3

So any common multiple must have prime factors of at least two 2s and two 3s.

$$\mathbf{36} = \underbrace{2 \times 2 \times \rlap{$\overbrace{}^{9}$}3}_{12} \times 3 \qquad \mathbf{72} = 2 \times \underbrace{2 \times 2 \times \rlap{$\overbrace{}^{9}$}3}_{12} \times 3$$

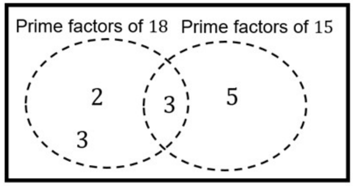
Task 2



18 , 36 , 54 , 72 , $\bigcirc\!\!\!\!\!\!\;90$, 108 ...
15 , 30 , 45 , 60 , 75 , $\bigcirc\!\!\!\!\!\!\;90$ ...

The methods show how it is possible to list numbers until a common multiple is found. In the top example, it is easy to see how common multiples will repeat. These are also useful with smaller numbers to see how common multiples relate to each other, and how they relate to the number line.

Where would this method become less useful? Imagine finding common multiples of 84 and 108?



$$90 = \underbrace{2 \times 3 \times \rlap{$\overbrace{}^{15}$}3}_{18} \times 5$$

These representations show the prime factors of two numbers and how they relate to the LCM. If prime factors are the building blocks of numbers, they are important to build multiples of those numbers.

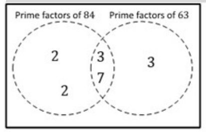
84 = 2 × 2 × 3 × 7 108 = 2 × 2 × 3 × 3 × 3 What would prime factors must all common multiples of 84 and 108 have?

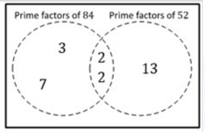
Exercise

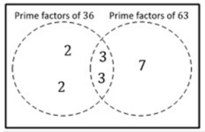
1.

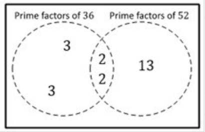
a) 63 = 3 × 3 × 7
b) 84 = 2 × 2 × 3 × 7
c) 52 = 2 × 2 × 13
d) 36 = 2 × 2 × 3 × 3

2.









3.

a) 81 d) 4
b) 4 e) 12
c) 9 f) 1

4.

The LCM is always 420, or the product of all the numbers.

Number 1	Number 2	LCM	Number 1	Number 2	LCM
140	12	420	60	70	420
84	20	420	210	28	420
60	28	420	140	42	420
210	12	420	84	70	420
84	30	420	60	105	420
60	42	420	84	105	420
210	20	420	140	105	420
140	30	420			

5.

There are many possible answers here. Firstly the two numbers must multiply to be greater that 1000, and they shouldn't share many prime factors. A simple way of doing it is choosing two prime numbers.

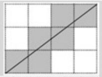
You can check you own answer here: https://www.omnicalculator.com/math/gcf-and-lcm

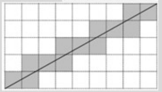
D1.

For any two integers, the product of the HCF and LCM is equal to the product of the original two numbers. Investigate the Venn diagrams to understand this further.

D2.

Some things to notice:
In many cases the number of squares it cuts through is 1 less than the sum of the sides. E.g. a 3×4 diagonal crosses 6 squares (3+4-1)
a 5×9 diagonal crosses 13 squares (5+9-1)





The only times this doesn't happen is if the two numbers have a common factor. This is because they cross at perfect corners. As is the case for a 4×6 diagonal